

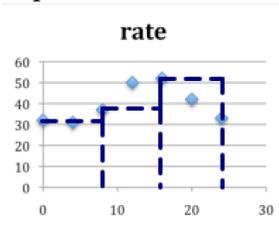
7.5 Numerical Integration

Group Work Target Practice

Ex 1:

Time (hrs)	0	4	8	12	16	20	24
Rate (barrels/hr)	32	31	37	50	52	42	33

Sketch the numerical method and then calculate an estimate of the total number of barrels pumped during the 24-hour periods for each of the following:

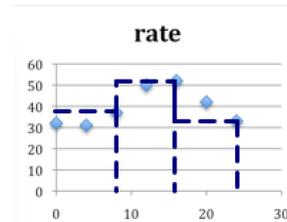


Left sum with 3 subintervals:

$$\Delta x = \frac{b-a}{n} = \frac{24-0}{3} = 8.$$

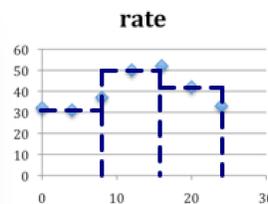
The left sum uses the left endpoint of each interval:

$$\sum_{i=1}^N f(x_{i-1})\Delta x = 8f(0) + 8f(8) + 8f(16) = 8 \cdot 32 + 8 \cdot 37 + 8 \cdot 52 = 968$$



The right sum uses the right endpoint of each interval:

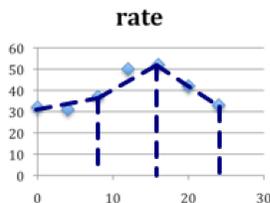
$$\sum_{i=1}^N f(x_i)\Delta x = 8f(8) + 8f(16) + 8f(24) = 8 \cdot 37 + 8 \cdot 52 + 8 \cdot 33 = 976$$



The midpoint sum uses the midpoint of each interval:

$$\sum_{i=1}^N f(x_{i-1} + x_i)\Delta x = 8f(4) + 8f(12) + 8f(20) = 8 \cdot 31 + 8 \cdot 50 + 8 \cdot 42 = 984$$

The trapezoid sum uses the line connecting the left and right endpoint of each interval:



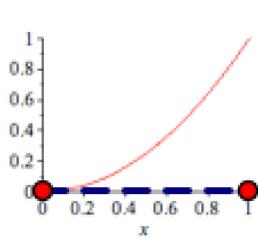
There is a shortcut:
$$\sum_{i=1}^N \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x = \frac{Left(3) + Right(3)}{2} = \frac{968 + 976}{2} = 972$$

Ex 2: You try it.

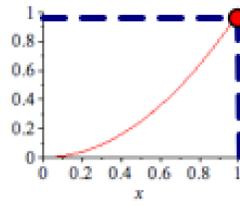
Ex 3: You try it.

Below are some graphs. For each of them, draw in the left, right, and trapezoid approximation with 1 subinterval over the interval $[0,1]$.

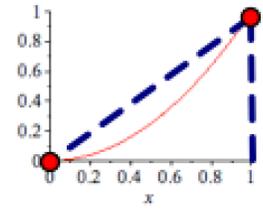
Here are the solutions for the first graph. You try the others.



Left(1):



Right(1):



Trap(1):

For each picture, determine if the left, right, and trapezoid approximations are overestimates or underestimates. Write OVER or UNDER in the blocks below.

Here is the solution for the first row. You try the others.

	Picture 1	Picture 2	Picture 3	Picture 4
Left Sum	UNDER	OVER	UNDER	OVER
Right Sum				
Trapezoid				

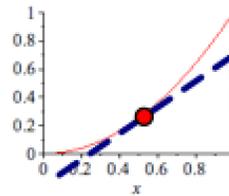
Determine what property of the function causes each of the approximations to be overestimates or underestimates.

Summarize your finding by completing the following statements:

Here is a solution for the first. You try the others.

The Left Sum is an underestimate of if the function is increasing and is an overestimate if is decreasing.

In the pictures below, draw in the tangent line at the midpoint and use that trapezoid (which gives the same area as the midpoint rectangle) to determine if the midpoint is an overestimate or underestimate for the area under each graph. Then fill in the following:



Here is a solution for the first. You try the others.