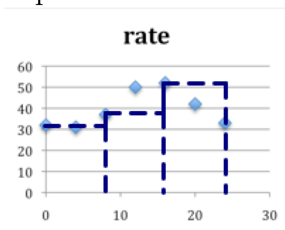
7.5 Numerical Integration
Group Work Target Practice

Ex 1:

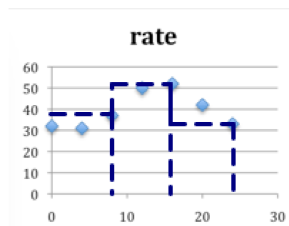
Time (hrs)	0	4	8	12	16	20	24
Rate (barrels/hr)	32	31	37	50	52	42	33

Sketch the numerical method and then calculate an estimate of the total number of barrels pumped during the 24-hour periods for each of the following:



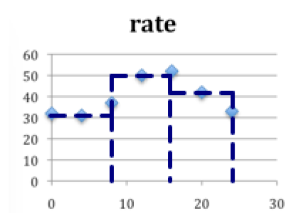
Left sum with 3 subintervals:
$\Delta x = \frac{b-a}{n} = \frac{24-0}{3} = 8.$
The left sum uses the left endpoint of each interval:

$$\sum_{i=1}^{N} f(x_{i-1})\Delta x = 8f(0) + 8f(8) + 8f(16) = 8 \cdot 32 + 8 \cdot 37 + 8 \cdot 52 = 968$$



The right sum uses the right endpoint of each interval:

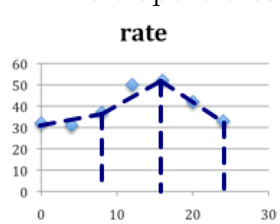
$$\sum_{i=1}^{N} f(x_i)\Delta x = 8f(8) + 8f(16) + 8f(24) = 8 \cdot 37 + 8 \cdot 52 + 8 \cdot 33 = 976$$



The midpoint sum uses the midpoint of each interval:

$$\sum_{i=1}^{N} f(x_{i-1} + x_i)\Delta x = 8f(4) + 8f(12) + 8f(20) = 8 \cdot 31 + 8 \cdot 50 + 8 \cdot 42 = 984$$

The trapezoid sum uses the line connecting the left and right endpoint of each interval:



There is a shortcut: $\displaystyle\sum_{i=1}^{N} \frac{f(x_{i-1}) + f(x_i)}{2}\Delta x = \frac{Left(3) + Right(3)}{2} = \frac{968 + 976}{2} = 972$

Ex 2: An artificial lake is created as a reservoir for agricultural irrigation. The rate at which the water flows out of the lake is regulated by a state agency. The table below gives the rate in hundreds of cubic feet per day at which the water is released every two days for 10 days.

Ex 1:

Time (days)	0	2	4	6	8	10	12
Rate (Hundreds of cubic feet per day)	120	110	100	80	50	20	15

Estimate the total amount of water that is released in that period using

A trapezoid sum with 6 subintervals. $\Delta x = \dfrac{b-a}{n} = \dfrac{12-0}{6} = 2$. So the width of the rectangles is 2. We calcuate the left(6) and the right(6) so we can average them to obtain trap(6):

left(6) : $f(0)2+f(2)2+f(4)2+f(6)2+f(8)2+f(10)2 = 120\times 2+110\times 2+100\times 2+80\times 2+50\times 2+20\times 2 = 960$

right(6) : $f(2)2+f(4)2+f(6)2+f(8)2+f(10)2+f(12)2 = 120\times 2+110\times 2+100\times 2+80\times 2+50\times 2+20\times +15\times 2 = 750$

Average these: $\frac{960+750}{2} = 855$

A midpoint sum with 3 subintervals. $\Delta x = \dfrac{b-a}{n} = \dfrac{12-0}{3} = 4$. So the width of the rectangles is 4, and the heights are function values evaluated at the midpoints:

$4f(2)+4f(6)+4f(10) = 4\times 110+4\times 80+4\times 20 = 840$

Is a midpoint sum with 6 intervals possible? No–midpoint values of the function have not been measured.

Ex 3: Estimate $\displaystyle\int_0^1 e^{-x^2}dx$ using

A left sum with 2 subintervals

$\Delta x = \dfrac{b-a}{n} = \dfrac{1-0}{2} = .5$. So the width of the rectangles is .5. The Riemann sum is:

$e^{-0^2}\times .5+e^{-.5^2}\times .5$

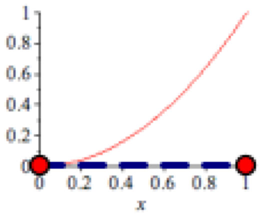
A right sum with 4 subintervals

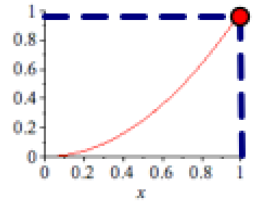
$\Delta x = \dfrac{b-a}{n} = \dfrac{1-0}{4} = .25$. So the width of the rectangles is .25. The Riemann sum is:

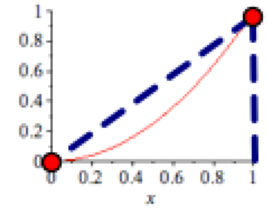
$e^{-.25^2}\times .25+e^{-.5^2}\times .25+e^{-.75^2}\times .25+e^{-1^2}\times .25$

Below are some graphs. For each of them, draw in the left, right, and trapezoid approximation with 1 subinterval over the interval [0,1].

Here are the solutions for the first graph.

Left(1): 

Right(1): 

Trap(1): 

You try the other graphs.

For each picture, determine if the left, right, and trapezoid approximations are overestimates or underestimates. Write OVER or UNDER in the blocks below.

	Picture 1	Picture 2	Picture 3	Picture 4
Left Sum	UNDER	OVER	UNDER	OVER
Right Sum	OVER	UNDER	OVER	UNDER
Trapezoid	OVER	UNDER	UNDER	OVER

Determine what property of the function causes each of the approximations to be overestimates or underestimates.

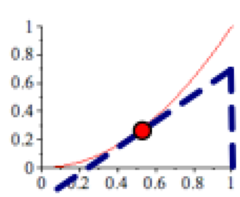
Summarize your finding by completing the following statements:

The Left Sum is an underestimate of if the function is increasing and is an overestimate if is decreasing.

The Right Sum is an underestimate of if the function is decreasing and is an overestimate if is increasing.

The Trapezoid Sum is an underestimate of if the function is concave down and is an overestimate if is concave up.

In the pictures below, draw in the tangent line at the midpoint and use that trapezoid (which gives the same area as the midpoint rectangle) to determine if the midpoint is an overestimate or underestimate for the area under each graph. Then fill in the following:



Here is a solution for the first. You try the others.

The Midpoint Sum is an underestimate of if the function is concave up and is an overestimate if is concave down.