9.1—9.3 Maple and Group Work Target Practice
9.1 Sequences in Maple

A *sequence of integers* is an ordered list of numbers, like the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21... You have explored sequences before, in middle and high school. In the 1120 setting we are interested in understanding the limit as $n \to \infty$—whether a sequence converges or diverges, among other concepts and real-life applications.

1. $a_n = ne^{-n} = \dfrac{n}{e^n}$
   (a) Write out (but do not simplify) the first two terms of the sequence.

   $n = 1$ _______________ $n = 2$ _______________

   (b) Does L'Hôpital's apply here? yes no Why or why not?

   (c) What is $\lim_{n\to\infty} a_n$? Show reasoning or work.

Open Maple. When you first launch it, there are icons. After choosing the **calculus** icon, at the bottom of the list open the **sequences** applet. (If that doesn't work, you can open it from the class highlights page). Be sure to check *Check for Convergence*.

Examine the last four sequences in the pull down menu as you investigate and answer:

2. $a_n = (2n + 1)^2$
   (a) The sequence converges to _______________ diverges

   (b) Sketch a rough plot of the sequence

   (c) (By-hand) Substitute n into the sequence and write out but do not simplify:

   $n = 1$ _______________ $n = 2$ _______________

3. $a_n = (\frac{-1}{2})^{n-1} - 1$
   (a) The sequence converges to _______________ diverges

   (b) Sketch a rough plot of the sequence

   (c) (By-hand) Substitute n into the sequence and write out but do not simplify:

   $n = 1$ _______________ $n = 2$ _______________

4. $a_n = \sin(\frac{1}{n^2}) + \cos(\frac{1}{n})$

   (a) The sequence converges to ______________ diverges

   (b) Sketch a rough plot of the sequence

5. $a_n = \frac{\cos(n)}{n}$

   (a) The sequence converges to ______________ diverges

   (b) Sketch a rough plot of the sequence

6. Which, if any of the sequences always increases ($a_n \leq a_{n+1}$ for all n)?

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\frac{1}{n^2}) + \cos(\frac{1}{n})$ $a_n = \frac{\cos(n)}{n}$ none

7. Which, if any of the sequences always decreases ($a_n \geq a_{n+1}$ for all n)?

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\frac{1}{n^2}) + \cos(\frac{1}{n})$ $a_n = \frac{\cos(n)}{n}$ none

8. A monotone sequence is one that always increases or always decreases, so circle the monotone sequences from your last two responses:

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\frac{1}{n^2}) + \cos(\frac{1}{n})$ $a_n = \frac{\cos(n)}{n}$ none

9. Are any of the sequences bounded (ie the a_n terms stay within a fixed region as $n \to \infty$, rather than approaching $\pm\infty$)? You can increase the number of terms in the sequence in the Maple applet.

   $a_n = (2n+1)^2$ $a_n = (\frac{-1}{2})^{n-1} - 1$ $a_n = \sin(\frac{1}{n^2}) + \cos(\frac{1}{n})$ $a_n = \frac{\cos(n)}{n}$ none

"A sequence per-se is inherently and literally primordial, and for me, this is a very alluring creative outlet. In other words, to be inventive in this medium, one must have mastery of only two things, numbers and ideas."
[Neil Sloane, founder of the Online Encyclopedia of Integer Sequences]