Series Theorems

- *Geometric Series*: If $\sum_{i=0}^{\infty} ax^i$ has x as a fixed ratio of successive terms and a constant then $\sum_{i=0}^{n} ax^i = \frac{a(1-x^{n+1})}{1-x}$ and so $\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x}$ provided $|x| < 1$ and diverges otherwise.

- *Terms not Going to 0*: If $\sum a_n$ has $\lim_{n->\infty} a_n \neq 0$, then the infinite series does not converge.

- *Linearity*: $\sum a_n + b_n$ converges to the sum of the individual series if both converge and diverges when only one diverges.

- *Integral Test*: If $\sum a_n$ has terms decreasing and $a_n > 0$ (eventually) and $\int_1^{\infty} a_n dn$ is known, then the series behaves the same way as it since $\int_a^{\infty} f(x)dx \leq \sum a_n \leq$ 1st term$+\int_a^{\infty} f(x)dx$.

- *Limit Comparison Test*: If $a_n > 0$ and $b_n > 0$ (eventually) and $0 < \lim_{n->\infty} \frac{a_n}{b_n} < \infty$, then $\sum a_n$ behaves the same way as $\sum b_n$.

- *Ratio Test*: For $\sum a_n$, if $\lim_{n->\infty} \frac{|a_{n+1}|}{|a_n|} = L$, then: $L < 1$ implies convergence; $L > 1$ implies divergence; $L = 1$ gives no information.

- *Alternating Series*: If we have an alternating series with $|a_n| \geq |a_{n+1}|$ and $\lim_{n->\infty} |a_n| = 0$, then the alternating series converges, and the truncation error of using $S_n \leq |a_{n+1}|$.

- *Taylor Series and Taylor Polynomial*: If $f(x)$ has continuous derivatives then it can be approximated near a by the series $\sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!}(x-a)^i$ or n^{th} degree polynomial $\sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$.

- *Taylor Polynomial Error*: The difference between the n^{th} degree Tayor polynomial and $f(x)$ evaluated at a value of x near a is at most $\left|\frac{M}{(n+1)!}(x-a)^{n+1}\right|$, where M is the upper bound on the absolute value of the $(n+1)^{st}$ derivative of $f(x)$ between x and a.