# MAT 1120 - Review for Final Exam

Here are some review questions in order to help you make connections among some of the different sections—also see prior quizzes and exams, class notes, *i-clicker* questions, Wiley and paper homework, and group work. These should be very familiar—these were mostly collated from prior questions in and outside of class whose solutions are online or should be in your notes, so that is where you should look to solutions. The whole point of the exam review is that I collated these for you in a form where it isn't attached to the answers unless you go back to the originals.

1. **For sequences** EXPLAIN or SHOW WORK documenting why your answer is correct:
   (a) write "sequence." does it converge or diverge, and why
   (b) what is the limit if it converges?
   (c) show work for L'Hôpital's Rule if it applies.

   **For series** EXPLAIN or SHOW WORK documenting why your answer is correct:
   (a) (LG 3) choose a series test we can successfully use on it from the Series Theorems sheet and write the name of the test
   (b) fully document why the series test works, including any assumptions
   (c) specify whether the series converges or diverges, and why

   (1) $s_n = e^n$

   (2) $s_n = e^{-n}$

   (3) $s_n = ne^{-n}$

   (4) $\displaystyle\sum_{n=0}^{\infty} e^n$

   (5) $\displaystyle\sum_{n=1}^{\infty} e^{-n}$

   (6) $\displaystyle\sum_{n=0}^{\infty} (\frac{1}{2})^n + (\frac{1}{e})^n$

   (7) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n} + (\frac{1}{2})^n$

   (8) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+6}$

   (9) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+5}$

   (10) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n}{n+5}$

   (11) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

   (12) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

   (13) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(14) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n \ln n}$

(15) $\sum_{n=0}^{\infty} \frac{n}{n!}$

2. What is the radius of convergence of $\sum_{n=1}^{\infty} \frac{(5x)^n}{n} = 5x + \frac{25x^2}{2} + \frac{125x^3}{3} + \ldots$. Show work, including naming a test from the Series Theorems sheet, and documenting why it works.

3. What is the interval of convergence of $\sum_{n=1}^{\infty} \frac{(5x)^n}{n} = 5x + \frac{25x^2}{2} + \frac{125x^3}{3} + \ldots$. Show work, including naming tests from the Series Theorems sheet, and documenting why they work.

4. Directly compute the Taylor polynomial of degree 4 for $\sin(x)$ about 1. Show work.

5. Directly compute the linear approximation of $\arctan(x)$ about 0.

6. Use the Lagrange Error Bound/Taylor's Inequality to find the maximum possible error in using the linear approximation about 0 to estimate $\arctan \frac{1}{5}$. Show work.

7. Sketch a graph of $\arctan(x)$ between 0 and $\frac{1}{5}$, sketch the linear approximation, and label the error.

8. If we use a higher degree Taylor polynomial, would it give us more or less error? Why?

9. Use reasoning rather than computation to determine what is the Taylor polynomial of $3 - 9x^2 + 18x^3$.

10. If a function is below the $x$-axis, increasing, and concave up

    a) Sketch the graph

    b) What are the signs of the coefficients in the second-degree Taylor polynomial?

    c) What, if anything, can we say about a point we might compute using Euler's method?

11. Compute the Taylor series for $\arctan(x)$ about 0. Show work.

12. Find the radius of convergence for the Taylor series for arctan(x) about 0. Show work, including naming tests from the Series Theorems sheets, and documenting why they work.

13. Use a known Taylor series to find the Taylor series about 0 for $\sin(x^2)$. Show work.

14. Use your last response and then calculate the first three terms of the Taylor series for $\int \sin(x^2)$. Show work.

15. Write a differential equation for the balance in an investment fund with time measured in years when the balance is losing value at a continuous rate of 6.5% per year, and payments are being made out of the fund at a continuous rate of $50,000 per year.

16. Write a differential equation for a cooling or heating scenario. Or for an object that grows proportional to its current amount.

17. Newton's law of cooling specifies that an object cools at a rate proportional to the difference between the object's temperature and room temperature. Say that we have an object where the constant of proportionality is -1, so that $\frac{dT}{dt} = -(T - 72)$, where 72° is the temperature of the room.

    a) First solve the differential equation by using the method of separation of variables. Show work.

b) Then apply the initial condition T(0) = 180 and show work to find $T(t)$.

c) Then check that your solution satisfies the differential equation.

d) Next sketch the slope field for the differential equation at a few grid points.

e) Use the DEplot to sketch a rough graphical solution.

f) Apply Euler's method one time with $\Delta x = .2$ starting at the point $(0, 180)$.

g) If we used a smaller $\Delta x$ would we have a better numerical estimate? Why or why not?

18. Look at the region $R$ between $y = \sin x$ and $y = 2 \sin x$ from 0 to $\pi$

    (a) Roughly sketch $R$ and a slice for the area of $R$
    (b) Set up the integral that gives the area and identify geometric/physical components
    (c) What is the main integration method we would use to integrate?
    (d) To the right of your sketch above, sketch a slice for the volume formed by revolving the same region $R$ about the $x$-axis.
    (e) Set up the integral that gives the volume and identify geometric/physical components

19. For various geometric problems, such as what is the work to pump water of 62.4 lb/ft$^3$ from a cylindrical tank on its side of radius 3, length 15, and buried 4 feet underground...

    (a) Sketch a diagram and a slice and label components
    (b) To the right of this sketch, sketch a diagram that shows why we need Pythagorean theorem or similar triangles and label components in terms of a given variable, say $y$ or $h$
    (c) What is the work for a slice
    (d) Set up an integral to find the total work
    (e) Identify the geometric and physical meaning of the components in the integral.
    (f) What is the main integration method we would use to integrate?

20. How would the previous problem change if you had a similar problem, but for a sphere?

21. How about a cone?

22. How about a pyramid?

23. How about a cylinder standing upright, like a garbage tank? Would you even need Pythagorean theorem or similar triangles?

24. What if you have the same problems or similar ones and want fo find the total mass rather than the work, like a rod of length 10cm has density $\frac{32}{x+3} gm/cm$ at a distance of $x$ $cm$ from the left.

    (a) Set up the integral that gives the mass and identify geometric/physical components
    (b) What is the main integration method we would use to integrate?

25. What is the arc length of the curve $\tan 3x^2$ from 0 to 1? Set up but do not evaluate.

26. 7 tons of pollutants are dumped once a day. If 25% are removed by natural process before the next dumping, then what is the quantity after 3 months? Set up as a series and then find the response using a series method. Show work.

27. Identify the kind(s) of series this is and explain what is wrong with the following statement.

28. Explain the mathematics in the visual or comic.

29. One of the four main educational goals at Appalachian is local to global perspectives, and it is also a theme in Calculus II and Analytic Geometry. Name an instance in our class where local perspectives where important in understanding the global perspective, and specify what is local and what is global in your example.

30. Another theme is in understanding infinity. Discuss an instance from our class in that context.

31. Another theme is approximation. Discuss an instance from our class in that context.

32. Identify which method from among w-subs, parts, partial fractions, trig sub, improper, calc 1, not elementary (ie approximation methods)

    a) $\int_0^1 e^{-x^2}\,dx$

    b) $\int xe^{-x^2}\,dx$

    c) $\int xe^{-x}dx$

    d) $\int \frac{x^2}{\sqrt{4-x^2}}dx$

    e) $\int \frac{x}{\sqrt{4-x^2}}dx$

    f) $\int \frac{3}{4-x}dx$

    g) $\int \frac{3}{\sqrt{4-x}}dx$

    h) $\int \frac{1}{4-x^2}dx$

    i) $\int \frac{1}{1+x^2}dx$

    j) $\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2(x)}dx$

    k) $\int_0^{\frac{\pi}{2}} \frac{1}{\cos^2(x)}dx$

| **Technique** | **What I want you to show me (don't integrate the final integral!)** |
|---|---|
| $w$-Sub | $w$, $dw$, and the integral with respect to $w$ |
| Parts | $u$, $u'$, $v$, $v'$, and $uv - \int u'vdx$ |
| Partial Fractions | the expansion, and the system of linear equations to solve for $A, B$ |
| Trig Sub | triangle pic, $x\&dx$, integral with respect to $\theta$ reduced |
| Improper | limit integral set up |

33. Evaluate the following integrals and show work and/or reasoning, including limits—but only if they apply. If numbers need to be plugged in you don't need to simplify, ie $\ln|5-3|$ or similar is ok here. If it is improper, do clarify whether the integral converges or diverges.