Multiplication of Matrices in 2.1 (Extension of 1.4)

Columns of B Method: $AB = \left[\begin{array}{ccc} A.\text{col}1B & \dots & A.\text{col}nB \end{array}\right]$

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}5 & 6 & 7\\8 & 9 & 10\end{bmatrix} = \left[\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}5\\8\end{bmatrix} \quad \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}6\\9\end{bmatrix} \quad \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}7\\10\end{bmatrix}\right]$$

$$= \begin{bmatrix}1\cdot5+2\cdot8 & 1\cdot6+2\cdot9 & 1\cdot7+2\cdot10\\3\cdot5+4\cdot8 & 3\cdot6+4\cdot9 & 3\cdot7+4\cdot10\end{bmatrix}$$

Dot Product Method: $AB_{ij} = \sum_{k=1}^{m} A_{ik}B_{kj}$ = [row$i$ A] · [column$j$ B]

$$= \begin{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix}\cdot\begin{bmatrix}5\\8\end{bmatrix} & \begin{bmatrix}1 & 2\end{bmatrix}\cdot\begin{bmatrix}6\\9\end{bmatrix} & \begin{bmatrix}1 & 2\end{bmatrix}\cdot\begin{bmatrix}7\\10\end{bmatrix}\\ \begin{bmatrix}3 & 4\end{bmatrix}\cdot\begin{bmatrix}5\\8\end{bmatrix} & \begin{bmatrix}3 & 4\end{bmatrix}\cdot\begin{bmatrix}6\\9\end{bmatrix} & \begin{bmatrix}3 & 4\end{bmatrix}\cdot\begin{bmatrix}7\\10\end{bmatrix}\end{bmatrix}$$

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Steps to solve $3x = 5 \rightarrow x = \frac{5}{3}$?

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Multiply both sides: $A^{-1}(A\vec{x}) = A^{-1}\vec{b}$

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Reduce identity: $\vec{x} = A^{-1}\vec{b}$

If A^{-1} exists then $A\vec{x} = \vec{b}$ has this unique solution for each $\vec{b}$ and $[A|\vec{b}] \to [I_{n\times n}|A^{-1}\vec{b}]$

Solving $A\vec{x} = \vec{b}$

Reduce $[A|\vec{b}]$ (0, 1 or ∞ sols) always works to solve $A\vec{x} = \vec{b}$.

If A is invertible, there is 1 sol: $\vec{x} = A^{-1}\vec{b}$ [$A \to I_{n\times n}$ so full pivots], so, (especially if we'll repeat for the same A but different b), $\vec{x}$ is multiplication of A^{-1} by $\vec{b}$:

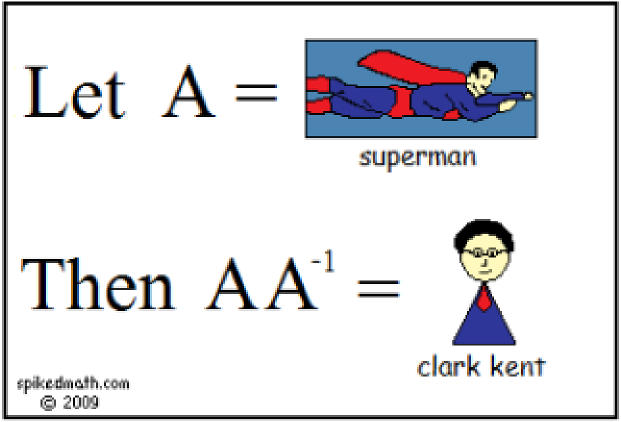
$$\begin{aligned} x - y &= -11 \\ 2x - y &= 3 \end{aligned} \qquad \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \text{ so}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \vec{b} = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -11 \\ 3 \end{bmatrix} = \ldots$$

(favored 1.4 method)

Use linear algebra to find the identity of superman.



http://spikedmath.com/042.html

Finding the Inverse

If A is invertible then $A \to I_{n\times n}$ and the same row operations that turn A to $I_{n\times n}$ turn $I_{n\times n}$ to A^{-1}.

$$[A|I] = \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \xrightarrow{r_2' = -\frac{c}{a}r_1 + r_2} \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & -\frac{bc}{a} + d & -\frac{c}{a} & 1 \end{array}\right]$$

$$\xrightarrow{r_2' = \frac{a}{ad-bc}r_2} \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$$\xrightarrow{r_1' = -br_2 + r_1} \left[\begin{array}{cc|cc} a & 0 & 1 + \frac{bc}{ad-bc} & -\frac{ab}{ad-bc} \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$$\xrightarrow{r_1' = \frac{1}{a}r_1} \left[\begin{array}{cc|cc} 1 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

so $A^{-1} = \frac{1}{ad-bc}\left[\begin{array}{cc} d & -b \\ -c & a \end{array}\right]$

Sherlock Holmes: *Elementary my dear Watson*

Finding the Inverse Via Elementary Row Operations

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Row operations can be written as matrix multiplications:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} . \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

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$E_p ... E_2 E_1 A = I$ so $A^{-1} = E_p ... E_2 E_1$