

## Spectrum/Spectral Analysis = the Eigenvalues

$$\begin{array}{ccccc} \vec{x} & \rightarrow & A\vec{x} & = & \lambda\vec{x} \\ \text{vector} & & \text{linear transformation of vector} & & \text{scalar} \times \text{eigenvector} \end{array}$$

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$f(x)$        $\rightarrow$        $\hat{O}f(x)$        $=$        $\lambda f(x)$   
function      linear operator on function      scalar  $\times$  eigenfunction

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**Ex 1: Schrödinger equation (quantum mechanics, chemistry)**

$\hat{O}$ : Hamiltonian operator (for a one dimensional particle)

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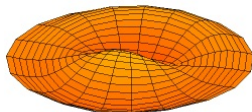
*The shape of a standing wave in a string fixed at its boundaries is an example of an eigenfunction of a differential operator. The admissible eigenvalues are governed by the length of the string and determine the frequency of oscillation. [Wikipedia]*

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## Ex 2: Spectrum of the Laplace operator

$\hat{O} = -\Delta$ : Laplace operator (divergence of gradient)



*This solution of the vibrating drum problem is, at any point in time, an eigenfunction of the Laplace operator on a disk*  
[Wikipedia]

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- Engineering: If frequency of the wind too close to the natural frequency of a bridge—oscillations.
- Linearized model: eigenvalue of smallest magnitude
- Tacoma Narrows Bridge collapse in 1940
- Design of car stereo systems to reduce vibration of the car due to music.

