Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

- matrix multiplication to scalar multiplication
- A keeps eigenvectors on the same line through $\vec{0}$ scaled by λ

To solve for eigenvectors of A, notice

Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

- matrix multiplication to scalar multiplication
- A keeps eigenvectors on the same line through $\vec{0}$ scaled by λ

To solve for eigenvectors of A, notice

$$A\vec{x} = \lambda\vec{x} = \lambda(I\vec{x}) = (\lambda I)\vec{x}$$
$$A\vec{x} - (\lambda I)\vec{x} = \vec{0}$$
$$(A - \lambda I)\vec{x} = \vec{0}$$

so we can solve for the nullspace of $(A - \lambda I)$. We want nontrivial solutions, so

Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

- matrix multiplication to scalar multiplication
- A keeps eigenvectors on the same line through $\vec{0}$ scaled by λ

To solve for eigenvectors of A, notice

$$A\vec{x} = \lambda\vec{x} = \lambda(I\vec{x}) = (\lambda I)\vec{x}$$
$$A\vec{x} - (\lambda I)\vec{x} = \vec{0}$$
$$(A - \lambda I)\vec{x} = \vec{0}$$

so we can solve for the nullspace of $(A - \lambda I)$. We want nontrivial solutions, so determinant $(A - \lambda I)$ = 0 let's us solve for any λs first.

Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

- matrix multiplication to scalar multiplication
- A keeps eigenvectors on the same line through $\vec{0}$ scaled by λ

To solve for eigenvectors of A, notice

$$A\vec{x} = \lambda\vec{x} = \lambda(I\vec{x}) = (\lambda I)\vec{x}$$
$$A\vec{x} - (\lambda I)\vec{x} = \vec{0}$$
$$(A - \lambda I)\vec{x} = \vec{0}$$

so we can solve for the nullspace of $(A - \lambda I)$. We want nontrivial solutions, so determinant $(A - \lambda I) = 0$ let's us solve for any λs first.

Example: $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Q1: Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ an eigenvector?

Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

- matrix multiplication to scalar multiplication
- A keeps eigenvectors on the same line through $\vec{0}$ scaled by λ

To solve for eigenvectors of A, notice

$$A\vec{x} = \lambda\vec{x} = \lambda(I\vec{x}) = (\lambda I)\vec{x}$$
$$A\vec{x} - (\lambda I)\vec{x} = \vec{0}$$
$$(A - \lambda I)\vec{x} = \vec{0}$$

so we can solve for the nullspace of $(A - \lambda I)$. We want nontrivial solutions, so determinant $(A - \lambda I) = 0$ let's us solve for any λs first.

Example: $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Q1: Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ an eigenvector?

Q2: All? $(A - \lambda I) = \begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}$.

Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

- matrix multiplication to scalar multiplication
- A keeps eigenvectors on the same line through $\vec{0}$ scaled by λ

To solve for eigenvectors of A, notice

$$A\vec{x} = \lambda\vec{x} = \lambda(I\vec{x}) = (\lambda I)\vec{x}$$
$$A\vec{x} - (\lambda I)\vec{x} = \vec{0}$$
$$(A - \lambda I)\vec{x} = \vec{0}$$

so we can solve for the nullspace of $(A - \lambda I)$. We want nontrivial solutions, so determinant $(A - \lambda I) = 0$ let's us solve for any λs first.

Example: $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Q1: Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ an eigenvector?

Q2: All? $(A - \lambda I) = \begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}$. First solve 0 = $\det(A - \lambda I) = \lambda^2 - 1$

Eigenvalues λ and Eigenvectors $\vec{x}$ of A: $A\vec{x} = \lambda\vec{x}$

Image citation: Shin Takahashi and Iroha Inoue *The Manga Guide to Linear Algebra*

- matrix multiplication to scalar multiplication
- A keeps eigenvectors on the same line through $\vec{0}$ scaled by λ

To solve for eigenvectors of A, notice

$$A\vec{x} = \lambda\vec{x} = \lambda(I\vec{x}) = (\lambda I)\vec{x}$$
$$A\vec{x} - (\lambda I)\vec{x} = \vec{0}$$
$$(A - \lambda I)\vec{x} = \vec{0}$$

so we can solve for the nullspace of $(A - \lambda I)$. We want nontrivial solutions, so determinant $(A - \lambda I) = 0$ let's us solve for any λs first.

Example: $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Q1: Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ an eigenvector?

Q2: All? $(A - \lambda I) = \begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}$.First solve 0 = $\det(A - \lambda I)$ = $\lambda^2 - 1$

Plug each λ in to solve for the nullspace of $(A - \lambda I)$. Augmented:

$\begin{bmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \end{bmatrix}$ reduce & parametrize. Geometrically: reflection.