Elementary Row Operations

- (Interchange) Swap two equations
- (Scaling) Multiply an equation by a non-zero constant
- (Replacement) Replace one row by the sum of itself and a multiple of another row [like $r_2' = -3r_1 + r_2$]

Gaussian Elimination (Echelon Form): 0s below the diagonal

- Save the 1st term in eq 1 and use it to eliminate all the other like-terms below it via $r_k' = cr_1 + r_k$
- Ignore eq 1 and use the 2nd term in eq 2 to eliminate all the like-terms below it.
- . . . 0s below the diagonal (interchange as needed)

Critical Analysis–Solutions

- [0 0 0 . . . 0 nonzero] inconsistent, 0 concurrent solutions.
- If consistent use pivots—any variables without pivots are free in a parametrization. Last row with a pivot (non-zero coefficients) will yield $x_k = ...$, and then solve for it and rest using back substitution and any free variables

Continuing to Gauss-Jordan/ReducedRowEchelon form: 0s or 1s on the main diagonal and 0 coefficients elsewhere

- Scale the last row with non-zero coefficients so that the diagonal entry is a 1.
- Use the last non-zero equation to eliminate the spots above it
- Repeat these steps using the second last non-zero equation.
- Read off the solutions from the pivots and the equal columns

$$\begin{bmatrix} 1 & -1 & -11 \\ 0 & 1 & 25 \\ 0 & 0 & 0 \end{bmatrix}$$ Gaussian or row echelon

$$\begin{bmatrix} 1 & 0 & 14 \\ 0 & 1 & 25 \\ 0 & 0 & 0 \end{bmatrix}$$ Gauss-Jordan or reduced row echelon