History of Integrals and Integration

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Definitions:

Integral: Relating to or concerned with mathematical integrals or integration.

Definite Integral: The difference between the values of the integral of a given function f(x) for an upper value b and a lower value a of the independent variable x.

Indefinite Integral: any function whose derivative is a given function.

Improper Integral: a definite integral whose region of integration is unbounded or includes a point at which the integrand is undefined or tends to infinity.

Integral Calculus: A branch of mathematics concerned with the theory and applications (as in the determination of lengths, areas, and volumes and in the solution of differential equations of integrals and integration.

Integration: a. the operation of finding a function whose differential is known b. The operation of solving a differential equation.

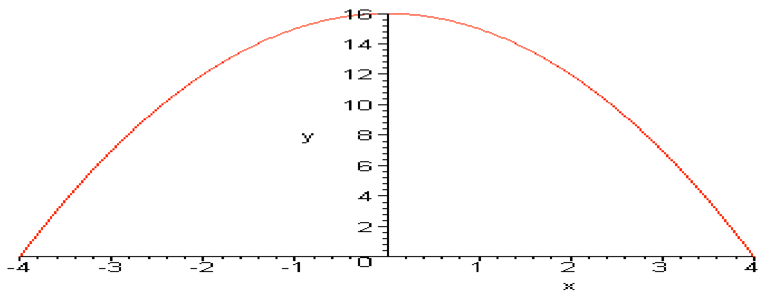
Method of Exhaustion: The method of exhaustion was an integral-like limiting process used by Archimedes to compute the area and volume of two-dimensional lamina and three-dimensional solids.

Method of Exhaustion:

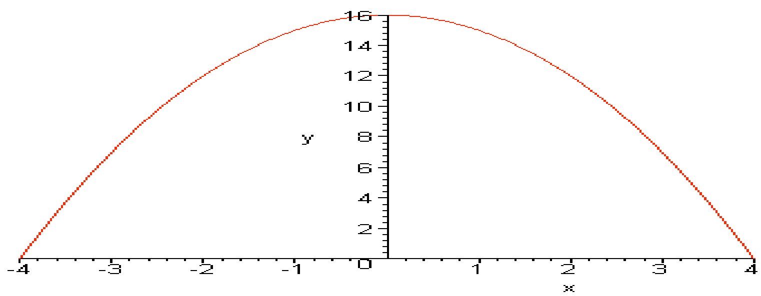
Archimedes, using Exodus's method of exhaustion, was able to find formulas for the surface areas and volumes of solids such as the sphere, the cone, and the paraboloid. Archimedes used various shapes that he already knew the area for, mostly triangles, and squeezed as many as he could under the curve until he found it "impossible" to fit anymore. With this method, Archimedes was able to come extremely close to the actual area.

Ex.

> Using the method of exhaustion, see if you can guesstimate the area under the parabola. You may choose any shape you know the area of, i.e. rectangle, triangle…. The only restriction is that you cannot go above the function, only under it!



What did you come up with?
Archimedes was able to prove that the area of a parboloid was 4/3 the area of the largest inscribed triangle possible. Try using Archimedes method and see what you get.

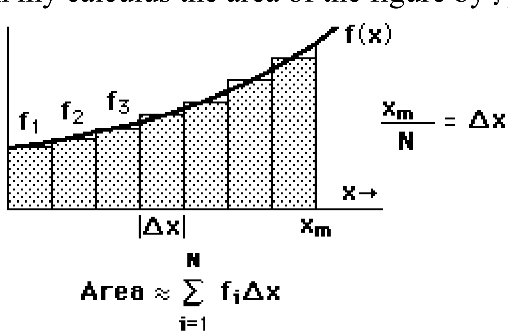


The Introduction of Calculus.

When Newton and Leibniz introduced Calculus, and more importantly the derivative and the anti-derivative, the study of integration became a lot more exact. Now mathematicians had the convenience of using analytical tools rather than geometrical tools to solve integrals. We also use can credit Leibniz with the symbols that we use to denote the integral. The term "Integral" was coined by Johann Bernoulli and published by his brother Jakob Bernoulli. Cauchy defined the integral of any continuous function on the interval [a,b] to be the limit of sums of the areas of thin rectangles. He wasn't able to prove this but did prove the Fundamental Theorem of Calculus. Dirichlet is responsible for the modern meaning of the term function. Riemann was able to prove Cauchy's definition of the integral to arbitrary functions on the interval [a,b].

Here is the basic idea behind modern integration:

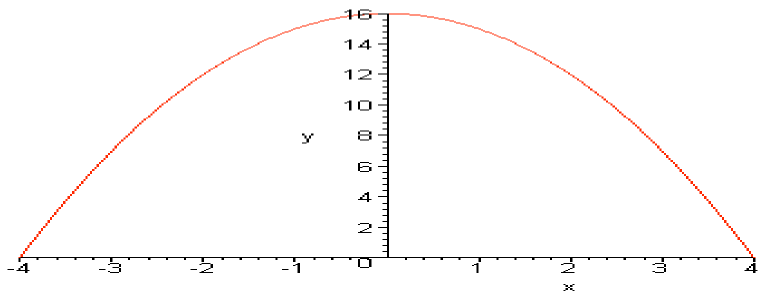
Leibniz: "I represent the area of a figure by the sum of all the [infinitesimal] rectangles contained by the ordinates and the differences of the abscissa … and thus I represent in my calculus the area of the figure by $\int y\,dx$."



The more rectangles you draw, the better the approximation becomes.

Ex.

Using rectangles approximate the area under the curve using 4, 8, 16 rectangles. Is there a big difference between the answers? How close are you to what you got earlier?



All right, you are almost done! You have just traveled through 2500 years of mathematics in calculating the area under a curve. All of this has lead up to Modern Integral Calculus! The parabola that you have been working on should be pretty familiar with most of you, f(x) = $-x^2 + 16$. So $\int_{-4}^{4} -x^2 + 16\,dx$, describes the area underneath the curve. Anti-differentiation with respect to x leads to: $-\frac{x^3}{3} + 16x\Big|_{-4}^{4}$. Plugging in the values for x, we get $(-\frac{4^3}{3} + 16*4) - (-\frac{-4^3}{3} + 16*(-4))$. This equates to 256/3 or ≈ 85.3333. Is this roughly the same answer you got? I hope so!