

Timeline For Euclidean Geometry and the “Fifth Postulate”

There is a lot of history of geometry before Euclid can onto the scene. The Babylonians, Egyptians, Greeks, and Pythagoreans did a great deal to advance the subject of geometry. The area of geometry that is taught most widely in high school mathematics classes is Euclidean Geometry. The focus for my project will be Euclidean Geometry.

~2000-500 BC- The Babylonians and Egyptians created many rules that became the basis for Geometry. They created rules, from experimentation, that were used only by the engineers within the civilizations. They had little previous knowledge on the subject of Geometry and both civilizations spent much time assuming rules and then confirming or refuting what they had declared to be true.

~500 BC- Pythagoras of Samos began to use logical thinking to make geometrical conclusions. He also used basic geometrical knowledge to make conclusions regarding advanced geometrical properties.

~300 BC (Before Euclid and *The Elements*)- Plato developed the precise idea of what a proof should entail. He also pushed for all conclusions, such as definitions and hypothesis, to be clear and able to be understood by most educated individuals.

~300 BC- Euclid wrote what is know as *The Elements*. This work is a collection of thirteen different books that are collection of geometric knowledge that was decided even before Euclid’s existence. This was knowledge expressed by Pythagoras, Plato, Hippocrates, and other mathematicians that were the pioneers in expressing geometrical conclusions. The first of these books was composed of 23 definitions, 5 postulates, 5 common notions, and 48 propositions.

~400 AD- Proclus wrote *Commentary* on Euclid’s *The Elements*. This commentary was written in an attempt to show that the fifth postulate could be derived from Euclid’s four previous postulates. He attempted to prove that Euclid’s Fifth Postulate could be derived from the other four postulates. He produced a proof, but it was later discovered that there were areas of the proof that were shown as being false.

1663- John Wallis attempted to prove Euclid's fifth postulate using the previous four postulates. Wallis thought that he had proven the fifth postulate, but he had just shown that the fifth postulate was comparable to: "To each triangle, there exists a similar triangle of arbitrary magnitude."¹

1697- Girolamo Saccheri produced a proof that some have said was the best attempt to date. What Saccheri did was to produce a contradiction of Euclid's fifth postulate to prove that the contradiction was false. He did this by using what is now called the "Saccheri quadrilateral." Saccheri's proof discusses the acute and obtuse angles of the Saccheri quadrilateral. His proof was reasonable for the obtuse angle, but when the acute angle came into play in the proof problems arose that prevented Saccheri from proving that the contradiction was false. Saccheri thought that he had found a contradiction, but instead he ended up reaffirming what Euclid had said was true.

1766- Johann Lambert also attempted to prove Euclid to be wrong. He went about his proof about the same way that Saccheri did and was also unsuccessful in his attempt.

1794- Adrien-Marie Legendre wrote *Eléments de géométrie*. Throughout this publication, Legendre had rewritten most of Euclid's propositions from *The Elements*. This rewritten text just simplified and reworded Euclid's previously stated propositions. Legendre also attempted to prove that Euclid's fifth postulate was also false. Through this attempt he ended up showing that the following was equivalent to Euclid's fifth postulate: "The sum of the angles of a triangle is equal to two right angles."² He was not successful in his attempt to prove that Euclid was wrong.

1817- Johann Gauss was certain that Euclid's fifth postulate could stand alone from the other four postulates. It is said that Gauss never published the work he produced pertaining to the fifth postulate and that he worked in secret.

¹ This information was found at http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Non-Euclidean_geometry.html.

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1825- János Bolyai, son of Farkas Bolyai, had become very interested in Euclid's fifth postulate, against his father's insistence that he distance himself from the problem. János wrote *strange new world*, an appendix to a book by his father, which showed all that János had studied for several years. He too had attempted to show that Euclid was false, but, yet again, he was unable to prove that Euclid's fifth postulate could not be a postulate.

1829- Nikolai Lobachevsky published an article in the *Kazan Messenger*, a Russian university publication. This article contained information pertaining to non-Euclidean geometry. Lobachevsky published numerous works dealing with hyperbolic and elliptic geometries. Hyperbolic geometry disproves Euclid's fifth postulate because there are more than one line that hold true when pertaining to the postulate. Elliptic geometry disproves Euclid's fifth postulate because there are no lines in an elliptic plane that hold true for Euclid's fifth postulate.

1840- Lobachevsky published another work. This work was called *Geometrical investigations on the theory of parallels*. In this work Lobachevsky discussed, in great detail, hyperbolic and elliptic geometries. He also revealed his conclusions that were drawn from his attempt to show that Euclid's fifth postulate could not be a postulate. His conclusion was just a replacement for the fifth postulate. His replacement for the fifth postulate was: "There exist two lines parallel to a given line through a given point not on the line."³

1868- The final attempt to prove Euclid's fifth postulate false was produced by Eugenio Beltrami. He wrote *Essay on the interpretation of non-Euclidean geometry*. This essay attempted to mesh 2-D non-Euclidean and 3-D Euclidean geometries. This "meshing" created a model that he based his argument that the fifth postulate was false on. Within this model Euclid's first four postulates were shown as true, but his fifth postulate was shown as false.

From the sources that I was using, I did find that after Beltrami, there was no noted work completed on the attempt to prove that Euclid's fifth postulate was false.

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