

Euclid's Fifth Postulate

Somewhere around 300 BC, a mathematician by the name of Euclid of Alexandria composed a mathematical work called Euclid's *Elements*. Euclid took the information that had been declared and proven by preceding mathematicians and put the information into one useable source. There was some information in *The Elements* is original to Euclid, but a lot of the information is information that Euclid found in other sources. *The Elements* gave Geometers one source that they could refer to when they were working on their research. *The Elements* were composed of thirteen books. The first of these books is what is going to be the most important to us. This first book is made up of twenty-three definitions, five common notions, and five postulates. The twenty-three definitions define terms such as line, plane, circle, and other common geometric terms. The five common notions are five statements that are seen as logical thoughts that as self-explanatory. The five postulates we will be focusing on are five statements that Euclid claimed were true. Euclid declared that there was no need to produce a proof to show that they were true. This is where the discrepancies lie. Most mathematicians had, and still have, no problems with the first four postulates being free standing in their truth. Yet the fifth postulate caused over 2000 years of debate. The fifth postulate, known as the Parallel Postulate is stated, from Euclid, as:

“If a straight line crossing two straight lines makes the interior angles on the same side less than the two right angles, the two straight lines, if extended indefinitely, meet in that side on which are the angles less than the two right angles.”

1) Construct a picture that visually describes what Euclid is trying to state in this postulate.

2) The language used in this postulate is archaic. Rewrite this postulate in more modern language. Make sure that you use your own words when rewriting this statement.

3) Proclus states that the Parallel Postulate was more of a theorem than a postulate. We have discussed the vocabulary used in geometry. What is the difference between a postulate and a theorem? Do not just state the definitions of postulate and theorem, but compare and contrast the two terms. You may state the definitions, but then you must point out the differences between a postulate and a theorem.

There were many attempts to prove that the Parallel postulate was not a postulate, but that it was actually a theorem. Euclid declared that postulates do not need to be proven because they are logical statements and logic can just be accepted. Many people, such as Saccheri (1697), Lambert (1755), Legendre (1794), and many others tried to prove that Euclid had made a mistake including this statement as one of his postulates. They were not satisfied that the fifth postulate was derived from logic and could, in turn, stand as a postulate. From these attempts there were many “equivalent” statements found and given for the parallel postulate. These statements may not have the exact same meaning as the fifth postulate, but they produce the same conclusions. These statements are:

1. There exists a pair of similar non-congruent triangles.
2. There exists a pair of straight lines everywhere equidistant from one another.
3. For any three non-collinear points, there exists a circle passing through them.
4. If three angles of a quadrilateral are right angles, then the fourth angle is also a right angle.
5. If a straight line intersects one of two parallels it will intersect the other.
6. Straight lines parallel to a third line are parallel to each other.
7. Two straight lines that intersect one another cannot be parallel to a third line.
8. There is no upper limit to the area of a triangle.

4) First, I want you to tell me “**your**” definition of equivalent. Second, I want you to take one of the statements above and tell me, in your own words, what the statement means (you can use drawings to help describe the statement. You must describe the drawing if you use one.). I do not want you to prove the statement, but I want you to restate the statement in words that your classmates can understand. Next I want you to tell me how your chosen statement is equivalent to the fifth postulate. **I want you to be as detailed as possible.**