HISTORY OF GROUP THEORY

A Brief Timeline of Important Developments

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1200s: The Moors demonstrated a knowledge of symmetry as evidenced by their artwork, particularly that of the Alhambra and its mosaics.

1400s: Leonardo da Vinci determined all possible symmetry groups of planar objects.

1700s: Muhammad al-Fullani al-Kishnawa evaluated the symmetries of a square to help in his study of magic squares.

1770: Joseph Lagrange studied permutations.

1799: Paulo Ruffini demonstrated the insolubility of the quintic equation using an argument based on groups of permutations. He also divided groups into cyclic or non-cyclic groups and simple or complex groups, though he did not use that terminology.

1801: Karl Gauss expanded previous works of Leonhard Euler and wrote about the orders of elements in a group and subgroups (also not using modern terminology).

1830: Evariste Galois first used the word *group* in its technical sense.

1832: Galois knew that normal subgroups were fundamental to the study of group theory. When the left cosets are equal to the right cosets of a group he said the group had been properly decomposed. He proved the non-Abelian simple group of smallest order was of order 60.

1843: William Hamilton invented the quaternions—a group consisting of eight elements: $G = \{\pm 1, \pm i, \pm j, \pm k\}$.

1844: Augustin-Louis Cauchy published his *Exercises d'analyse et de physique mathématique* in which he proved what is now commonly known as Cauchy's Theorem: Each group whose order is divisible by a prime p must have at least one subgroup of order p. He also used cyclic notation and discussed the order of permutations.

1846: In a paper published posthumously, Galois determined the solvability of equations based on substitution groups.

1848: Joseph Serret taught Group theory as a course in Paris.

1854: Arthur Cayley founded the theory of abstract groups. Previously groups had only been studied as an application to other branches of mathematics; Cayley made group theory its own subject apart from other branches of mathematics.

1858: Leopold Kronecker proved (in modern terms) the Fundamental Theorem of Finite Abelian Groups: Every finite Abelian group is a direct product of cyclic groups of prime-power order. Also in this year, the Institute of France offered a prize to be rewarded for outstanding research in the field of group theory. It was never awarded but sparked much new study.

1859: Cayley demonstrated that the quaternions were a group of order eight under multiplication.

1866: Group theory was first mentioned in a textbook—the 3rd edition of Serret's *Algèbre*.

1870: Camille Jordan published his *Traité des Substitutions* (totally devoted to group theory) in which he laid out the importance of isomorphisms of permutation groups. Also, Kronecker gave a formal definition of an abstract group.

1872: Ludvig Sylow proved the important theorem known by his name that expanded on the previous results of Cauchy's Theorem: Given a finite group G and a prime p, if p^k divides the order of G, then G has at least one subgroup of order p^k.

1882: Heinrich Weber called a group Abelian if it had the commutative property under multiplication. He also gave the modern axiomatic definitions for a group:

A system G of h arbitrary elements $\theta_1, \theta_2, \ldots, \theta_h$ is called a group of degree h if it satisfies the following conditions:

I. By some rule which is designated as composition or multiplication, from any two elements of the same system one derives a new element of the same system. In symbols $\theta_r \theta_s = \theta_t$.

II. It is always true that $(\theta_r \theta_s)\theta_t = \theta_r(\theta_s \theta_t) = \theta_r \theta_s \theta_t$.

III. From $\theta\theta_r = \theta\theta_s$ or $\theta_r\theta = \theta_s\theta$ it follows that $\theta_r = \theta_s$. (Burton, 2003, p. 602)

1896: Weber published *Lehrbuch der Algebra*, which became the standard text on group theory for many years.

1916: Henry Stager published *A Sylow Factor Table for the first Twelve Thousand Numbers*.

1960s: The Feit-Thompson Theorem proved that a non-Abelian simple group must have even order.

1976: Ron Rivest, Adi Shamir, and Leonard Adleman began work on RSA Coding that is based on properties of group theory.

1980: Robert Griess constructed the largest of sporadic simple groups. Its order is approximately $8x10^{53}$.

1981: Daniel Gorenstein, speaking for a team of mathematicians, declared that all finite simple groups had been classified. The proof that there are no more is over 10,000 pages long.

Selected Bibliography

Burton, D.M. (2003). *The History of Mathematics: An Introduction* (5th ed.). New York: McGraw-Hill.

Cajori, F. (1991). *A History of Mathematics* (5th ed.). New York: Chelsea.

Class notes: History of Mathematics. Spring 2003.

Gallian, J.A. (2002). *Contemporary Abstract Algebra* (5th ed.). New York: Houghton Mifflin.

Greenwald, S. The symmetries of a Square [Electronic Version]. Retrieved April 30, 2003 from http://www.cs.appstate.edu/~sjg/class/3010/square.html

Katz, V.J. (1998). *A History of Mathematics: An Introduction* (2nd ed.). New York: Addison-Wesley.

O'Connor, J.J. & E.F. Robertson. The development of group theory [Electronic version]. Retrieved April 16, 2003, from http://turnbull.mcs.st-nd.uk/~history/HistTopics/Development_group_theory.html