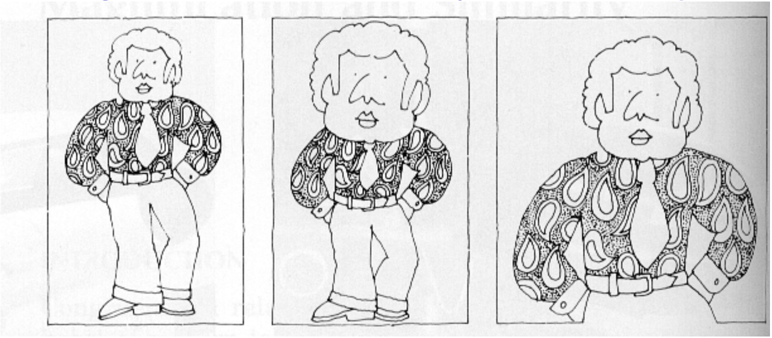
Congruence and Similarity: Same Shape?



congruence

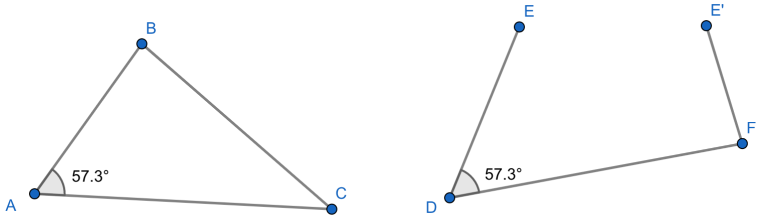
corresponding sides congruent and corresponding angles congruent

similarity

corresponding sides proportional and corresponding angles congruent

Congruence and Similarity: Euclidean SAS

- SAS: Two sides of the broken triangle *DEF* are the same length as the corresponding sides in $\triangle ABC$ ($\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$). The included angle is congruent ($\angle BAC \cong \angle EDF$). How many triangles can we create? Must they be congruent? If not, must they be similar? Why or why not?

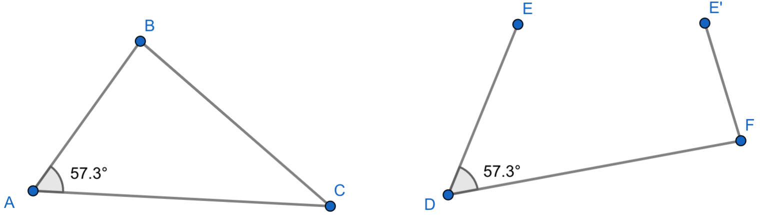


https://www.geogebra.org/geometry/apkxfb5y

Congruence and Similarity: Proof of SAS

http://www.atheistrepublic.com/sites/default/files/styles/blog-featured-image/public/proof.jpg

Congruence and Similarity: Proof of SAS



Let $\triangle ABC$ and $\triangle DEF$ satisfy SAS with $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle BAC \cong \angle EDF$. To show congruence, superimpose the triangles so that A is on D and $\overline{AB}$ is on $\overline{DE}$.

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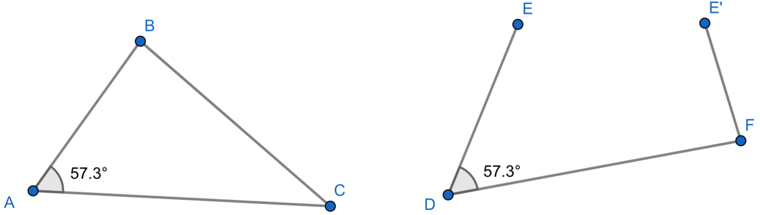

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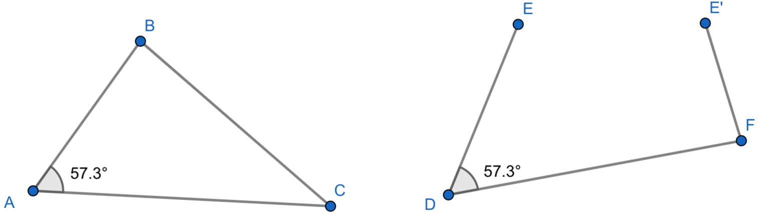

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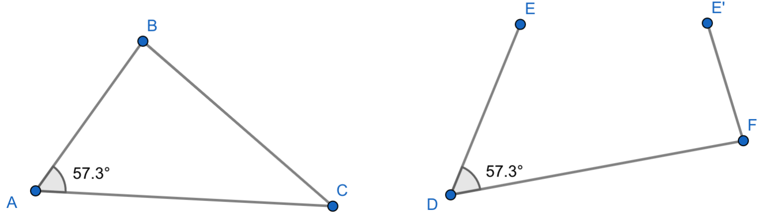
Congruence and Similarity: Proof of SAS



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Linear Transformations of the Plane

Rotation: $\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$ Dilation: $\begin{bmatrix}c & 0\\ 0 & c\end{bmatrix}$ Horizontal Shear: $\begin{bmatrix}1 & k\\ 0 & 1\end{bmatrix}$

Projections: y=x line: $\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ x-axis: $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ y-axis: $\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

Reflections: y=x line: $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ x-axis: $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ y-axis: $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$

Translation: $\begin{bmatrix}1 & 0 & h\\ 0 & 1 & k\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix} = \begin{bmatrix}x+h\\ y+k\\ 1\end{bmatrix}$ Others: $\begin{bmatrix}a & b & 0\\ c & d & 0\\ 0 & 0 & 1\end{bmatrix}$

Isometries

isometry: mapping of a metric space onto itself that preserves distance between points
plane: translations, rotations, reflections, and glide reflections
$\rightarrow$ congruence

What preserves shape but not congruence from among dilation, shear, projection?

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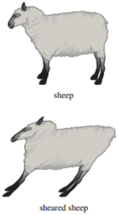
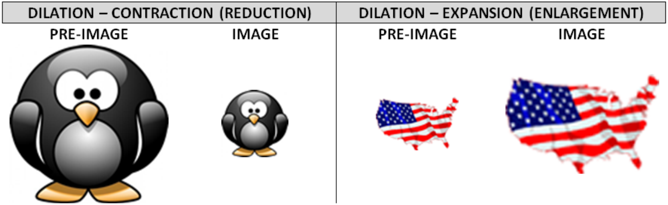


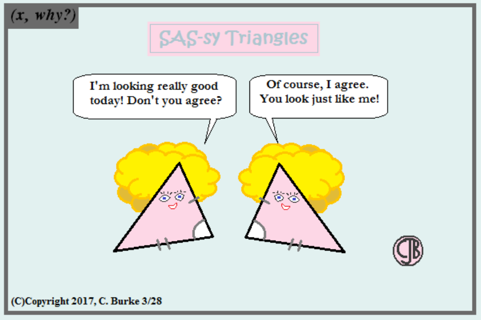


Linear Algebra and Its Applications by David C. Lay,

`http://www.geometrycommoncore.com/content/unit2/gsrt1/teachernotes1.html`

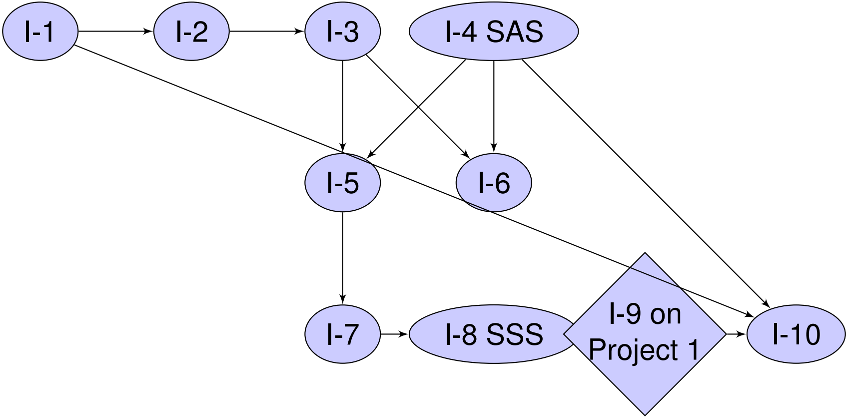
SAS in Other Contexts

- transformations
  https://www.geogebra.org/m/bM5FkyFK
- analytic/metric: Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$
- other axiom systems: SAS triangle congruence is a postulate not a proposition, or it is proven from one of the other congruence theorems



Propositions, Assumptions and Applications

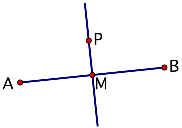
Proof Considerations: I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.



SAS Application: Perpendicular Bisectors Equidistance

Sketch of Proof:

- Proposition 10 constructs midpoint
- Proposition 11 lets us construct a perpendicular through a point on a line creating two right angles
- Select any point on the perpendicular bisector



- Now $\overline{PM} \cong \overline{PM}$ by CN 4 and $\overline{AM} \cong \overline{MB}$ by midpoint
- By Postulate 4, all right angles are equal so if we use postulate 1 to construct $\overline{AP}$ and $\overline{PB}$, then we have SAS in $\triangle PMA$ and $\triangle PMB$.
- By Proposition 4, $\triangle PMA \cong \triangle PMB$ and so $\overline{AP} \cong \overline{PB}$

Extension: Circumcenter

- Look at the intersection of 2 perpendicular bisectors of $\triangle ABC$, say the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$.

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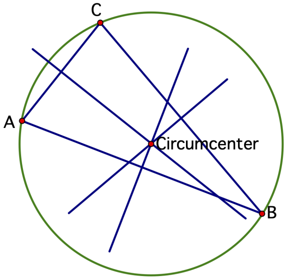
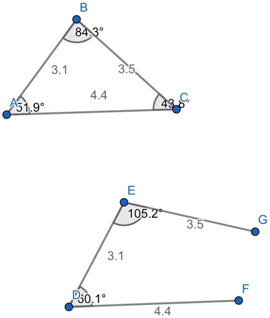
- Look at the intersection of 2 perpendicular bisectors of $\triangle ABC$, say the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$.
- It must be equidistant to all 3 vertices of the triangle.



- The 3rd perpendicular bisector contains all the points that are equidistant to 2 of the vertices so the intersection is on it too.
- All 3 perpendicular bisectors intersect at the circumcenter.

Congruence and Similarity: Euclidean SSS

- SSS: The three sides of the broken triangle are the same length as the corresponding sides in triangle ABC. How many triangles can we create? Must they be congruent? If not, must they be similar? Why or why not?



https://www.geogebra.org/geometry/bw4eagnh

Congruence and Similarity: Proof of SSS

- Superimpose
- I-7
- CN 4

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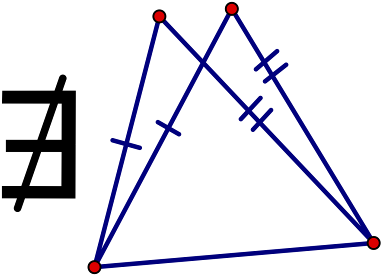
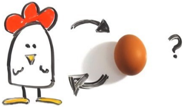
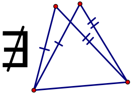


SSS in Other Contexts

- transformations
  https://www.geogebra.org/m/fskReBGs
- analytic/metric: Law of Cosines: solve for C in $c^2 = a^2 + b^2 - 2ab\cos C$
- other axiom systems: SSS is a postulate not a proposition and is used to prove SAS
- Hadamard's argument:
  - Move triangle so that $\overline{BC}$ sits on $\overline{B'C'}$, both triangles sit on the same side, and assume for contradiction that A is not on A'
  - B is the same distance from A and A' so it lies on the perpendicular bisector of $\overline{AA'}$. C is similar, so $\overline{BC}$ is the perpendicular bisector.

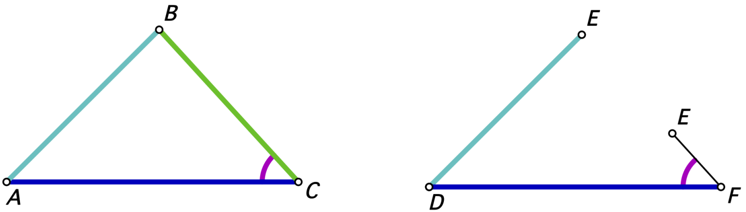




www.brainpickings.org/2013/02/01/which-came-first-the-chicken-or-the-egg/

Congruence and Similarity: Euclidean SSA

- SSA: Two sides of the broken triangle *DEF* are the same length as the corresponding sides in $\triangle ABC$ ($\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$) . An opposite angle is congruent ($\angle ACB \cong \angle DFE$). How many triangles can we create? Must they be congruent? If not, must they be similar? Why or why not?



https://www.geogebra.org/m/E8cZQwjV

Congruence and Similarity: Proof of (not) SSA