

Applications of Congruence and Similarity

- For triangles, SSS and more are lists of possible design choices in Euclidean geometry of the plane.
- Every isometry in the plane is a product of at most three reflections (via SSS)
- Two isometries which agree on three non-collinear points are the same on all points (via SSS)
- More generally, in other geometries, or for more points, like for CAD or CAGD programming, when are design choices complete or when do we need more choices?
- Many additional real-life applications

Euclidean Quadrilateral Design Choices

Consider what is the smallest amount of information we need to determine that quadrilaterals are congruent or similar?

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Euclidean Quadrilateral Design Choices

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- AA triangles

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Euclidean Quadrilateral Design Choices

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- **SSSS quadrilaterals**

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- **AA triangles**

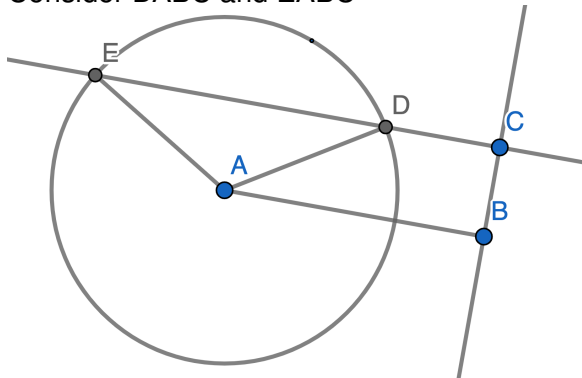
<https://www.geogebra.org/m/Q8EYTUK2>

- **AAA quadrilaterals**

<https://www.geogebra.org/m/uKnpbCCc>

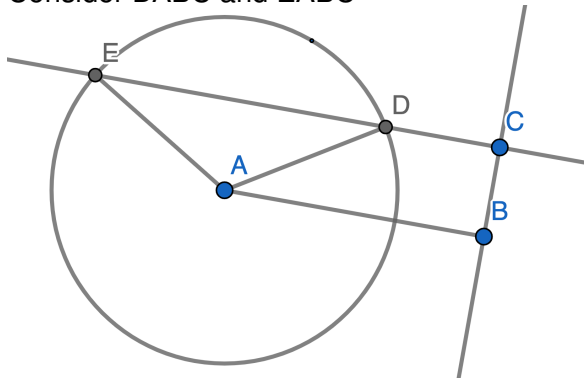
SSASA Similarity for a Euclidean Quadrilateral?

Consider $DABC$ and $EABC$



SSASA Similarity for a Euclidean Quadrilateral?

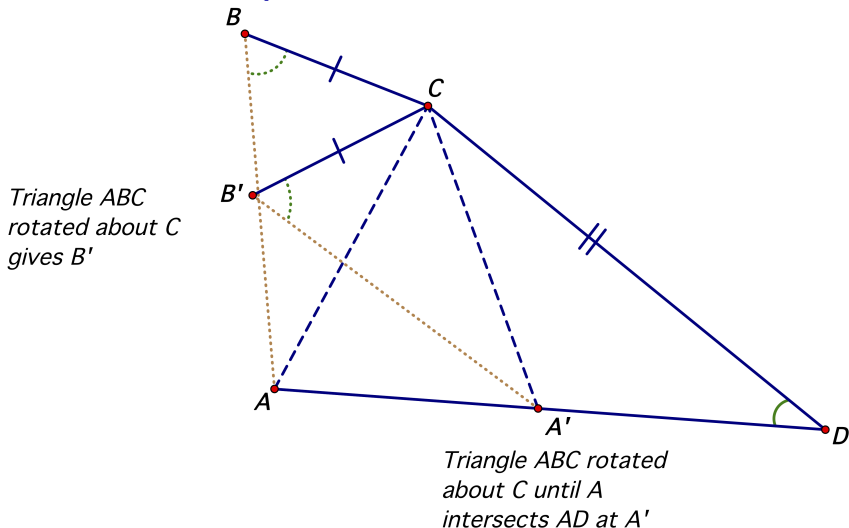
Consider $DABC$ and $EABC$



3 congruent sides and 2 right angles
implications for 4 pieces of information?

ASSA Similarity for a Euclidean Quadrilateral?

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Geometric Modeling Using Similarity

geometric similarity is common in deriving and testing physical and biological relationships

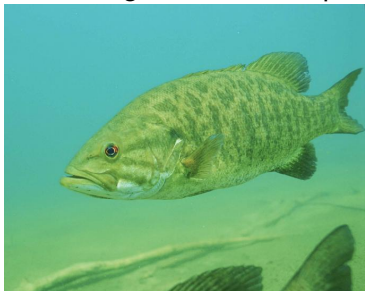


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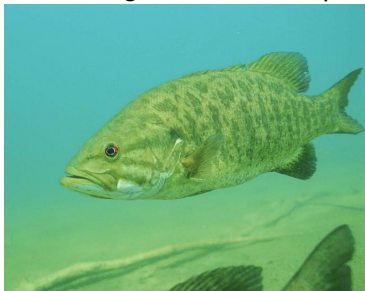


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a species of bass? $\frac{\text{length}}{\text{height}}$

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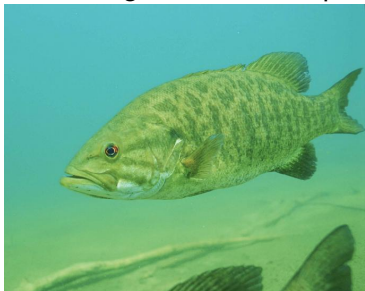


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humans? $\frac{\text{armspan}}{\text{height}}$ i.e. $l \propto h$.

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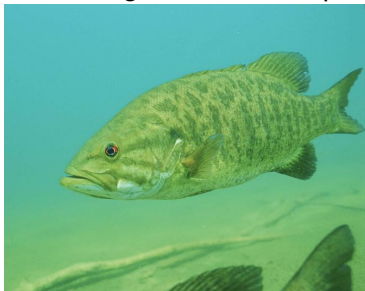


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volume?

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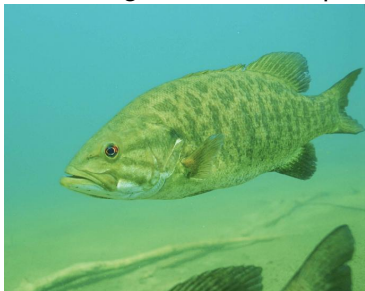


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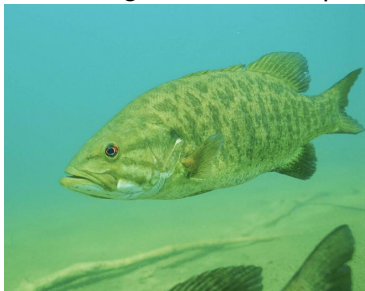


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surface area?

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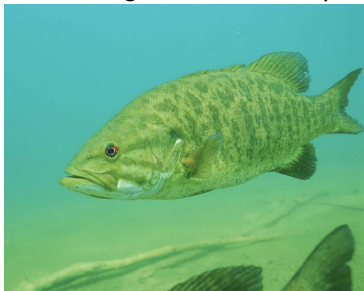


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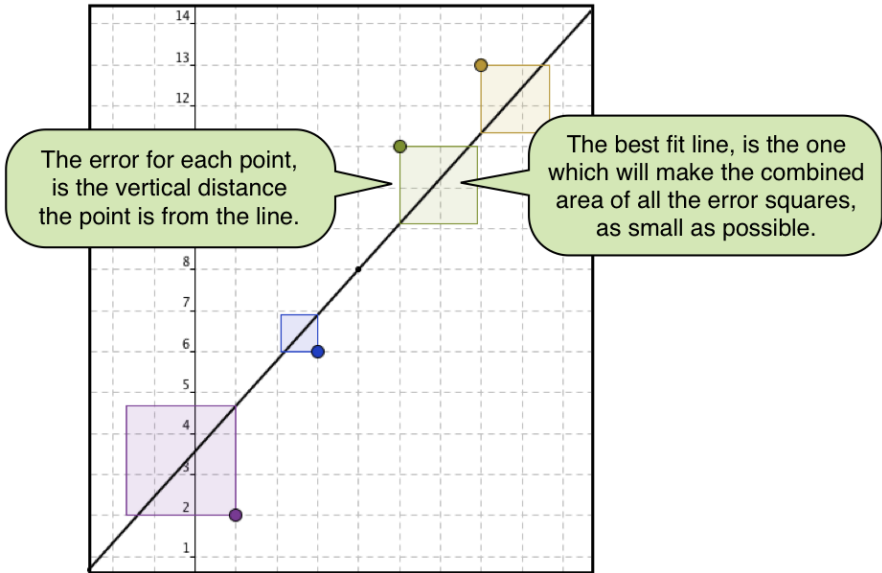
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volume? $v \propto l^3$

surface area? $a \propto l^2$

weight = volume \times average weight density, so if that density is constant, then $w \propto l^3$



<http://math.maine121.org/welcome/chapter-5/>

Geometric Modeling Using Similarity

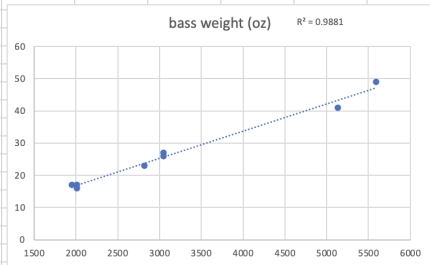
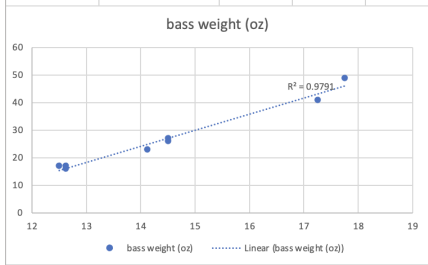
geomodel.xls presents data from the winners of the 1976 Olympics by their weight lifting class

- $\frac{\text{lift}}{\text{weight}}$ = constant slope, i.e. lift \propto weight?
- Click on the letters B and C above the relevant columns, using the command key, so that both columns are highlighted.
- Under Insert, look for a *Scatter* or *Marked Scatter* chart.
- Once the chart appears, control click on one of the points on the Chart, so that the menu is brought up which lets you *Add Trendline*.
- Click on *Display R-squared value on chart*. R^2 measures the y-values distances via a sum of squares as variation in the dependent variable explained by linearity and the closer to 100% the stronger the relationship.

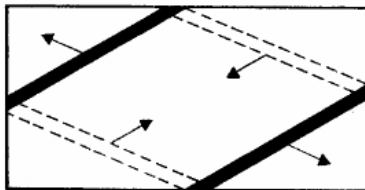
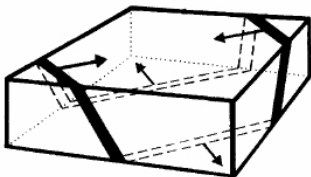
Geometric Modeling Using Similarity

basssimilarity.xlsx presents data on bass length (in) and bass weight (oz)

bass length (in)	bass length^3 (in^3)	bass weight (oz)
14.5	3048.625	27
12.5	1953.125	17
17.25	5132.953125	41
14.5	3048.625	26
12.625	2012.306641	17
17.75	5592.359375	49
14.125	2818.158203	23
12.625	2012.306641	16



Sliding a Ribbon Off a Box Using Similarity



The Geometric Viewpoint: a Survey of Geometries by Thomas Sibley