MAT 3610 Video Interactions

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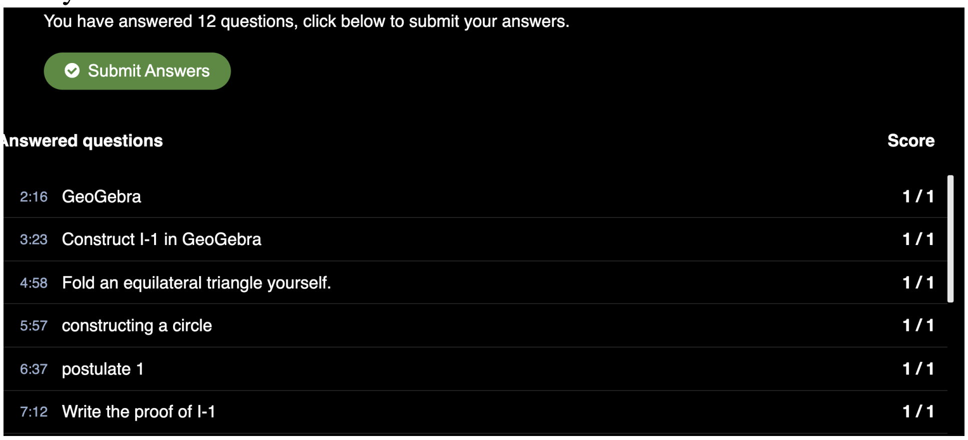
These are interactions in some of the interactive videos I created. In the interactive video, the video pauses, asks a question, and requires a response to proceed. To earn credit we watch the entire video and submit the correct answers via the green "Submit Answers" button at the very end of the video, the one that shows all the questions we have answered—we use the check feature on interactive questions in order to help and can redo the responses until they are correct.



Each video includes directions "to take notes on that you can bring with you to class: pause regularly to take notes that you can bring with you to class especially on concepts, proofs, Interactive Geometry Software and other visualizations, and any remaining questions." Above each video we include resources needed such as GeoGebra exploration links and Euclid's *Elements* Book 1.

axiomatic systems and constructions 1 interactive video

- Write instructions for assembling a peanut butter and jelly sandwich that a robot could understand and follow. Assume you have the needed materials already.
- How do we get completion credit for videos?
- Individual questions are repeatable and videos continue to be available later and are repeatable too. Which are true about your own notes on videos?
- If you've never played Minesweeper before, try a beginner game.
- Write out a proof that B1 is a number. Be sure to refer to the Axioms. Also make use of C1=1 and C2=mine.
- Make sure that your proof has both axioms in it and consider improvements that would help the robot follow the proof and be convinced by it.
- Write the axioms of incidence geometry.
- Consider how to prove the statement that there exist at least three distinct lines not all through any common intersection point.
- This proves that we have at least three distinct lines, so now consider how to prove that they don't share a common intersection point.
- Write out a proof in your notes.
- Sketch an incidence geometry with exactly 3 points that satisfies all the axioms.
- Consider how this Fano model with 7 points also satisfies the axioms of incidence geometry.

- Why isn't incidence geometry complete?
- Take a look at the definitions in Euclid's Elements, especially definitions 1, 2 and 4, either on the first page of the handout, if you have it, or online otherwise, at the link I put above this video.
- Next, take a look at Euclid's First 3 Postulates, the axioms, either on the back of the first page of the handout, if you have it, or online otherwise.
- In your notes, roughly sketch visualizations of Euclid's Postulates 1, 2, and 3.
- Using the quadrilateral I handed to you, or by making one yourself, fold so that A and B overlap and the lines emanating from them do to. Then pinch fold at the midpoint.
- Pinch fold the other 3 midpoints.
- What do the 4 segments connecting adjacent midpoints form?
- Open up GeoGebra Geometry. You can use one of the links from above the video.
- Use the segment tool to create a quadrilateral and get practice working in the IGS (Interactive Geometry Software).
- Under Construct, use the "Midpoint or Center" to construct the four midpoints.
- Use the segment tool to connect adjacent midpoints.
- Under Measure, use Angle and select 3 points at a time to measure each angle in what seems to be the parallelogram. formed by the midpoints.
- Write some benefits and challenges of IGS (Interactive Geometry Software) and roughly sketch the construction in your notes.

3610 course intro interactive video

- What should you call me in class and in your electronic salutations (like the forum)?
- Is a growth mindset an important part of class?
- Open up course calendar above the video.
- Consider the deadlines and activities and assignments on the PDF calendar.
- Notice that as shown in the picture of our classroom, tables of four people are working together during class and it will help if some of you have a laptop, tablet, or phone you can bring to classes to make use of Geometry GeoGebra and more.
- Consider the welcoming and active learning environment during class.
- Even though we aren't meeting for the third hour, its time is built into the activities. The university recommends 2-3 hours outside of class for each credit hour (c.h.) and we add the 3rd hours time that we aren't meeting in too: that gives 2*3c.h. + 1 to 3*3c.h. + 1 as weekly engagement time outside of our synchronous sessions. Since there are two of those, divide these ranges total by 2 to see the university recommended hours between each class.
- Where do you we find the checkmarks for completion?

- Are video notes important?
- How do you earn a checkmark for interactive videos?
- Consider how you will create PDFs of your worksheet responses and sketches.
- Even with a checkmark for completion, you might have some aspects that need improvement on a worksheet. Where do you go to see my feedback?
- Which boxes on the right do you click on to self-report completion rather than earning it via receiving a proficient grade or when you access an assignment?
- Consider the 6 projects and their due dates in the PDF calendar from above the video.
- How do you achieve completion in a reflection?
- Many items have strict deadlines just before class, including begins, projects, reflections, and worksheets. Try to attempt videos for completion and take video notes by the listed date whenever possible as the material builds on itself but those have second chances. Consider the grading percentages listed here and the deadlines on the PDF calendar.
- Consider the general skills from the prerequisite of Techniques of Proof and its prerequisite of Calculus II with Analytic Geometry that we need in 3610 and consider whether there are any you'll want to review (mathlab or online sources are great for review).
- Where can you go for help outside of class?
- What is some common advice from prior students?

axiomatic systems and constructions 2 interactive video

- Download or open the GeoGebra Geometry App at `https://www.geogebra.org/download` or `GeoGebraGeometryathttps://www.geogebra.org/geometry`
- Test out the equilateral triangle construction in GeoGebra. You can rewind the video as needed.
- Fold an equilateral triangle yourself. You can rewind the video.
- In the construction we create a circle of radius AB and center A. What postulate lets us construct this circle?
- What postulate lets us construct the lines?
- Write out the proof of I-1 in your notes to help you internalize it. You can rewind the video.
- Fold a perpendicular yourself. You can rewind the video.
- Where did D go?
- What is the difference between Postulate 1 and Proposition 1?
- Write out the proof of I-11 in your notes to help you internalize it (the last 2 slides). You can rewind the video.
- What is logically equivalent to the statement of I-11?
- Where does G first arise from in this construction of I-11?

congruence and similarity 1 interactive video

- Write the definitions of congruence and similarity in your notes. Be sure that you take notes for videos in a way that you can bring with you to future classes.
- Open the SAS GeoGebra above the video and leave E alone but drag E' to complete the triangle.
- What does SAS in a Euclidean triangle lead to?
- Write out the proof that SAS leads to congruency on something you can bring to classes as you consider limitations and underlying assumptions.
- Which is true regarding a limitation of this proof relating to the underlying assumption of BC and EF?
- Consider what transformations of the plane preserve congruence of figures, i.e. are isometries
- Sketch a rough picture of the sheared sheep and label it as a shear
- Consider what transformations of the plane preserve similarity but not congruence
- Open and explore the SAS transformations GeoGebra by Tim Brzezinski above the video
- Read over Postulates 1 and 4 and Propositions 4, 10 and 11 in Euclid's Elements.
- Write a paragraph proof in your notes, including an introduction, sentence connectors, and a conclusion.
- Sketch a rough picture of the intersection of the perpendicular bisectors of AB and AC for triangle ABC in your notes and consider what you can say about it
- In your notes, write out reasoning that the 3 perpendicular bisectors of a triangle intersect at one point, the circumcenter, the center of a circle going through the 3 vertices, but do not write a complete proof.
- Look for 2 places to complete the triangle in the SSS GeoGebra above the video and sketch them in your notes.
- Read Proposition 7 in *Euclid's Elements*.
- Open and explore the SSS transformations GeoGebra by Tim Brzezinski above the video
- If $a = 3$, $b = 4$, and $c = 2$, use the law of cosines to solve for C, the angle opposite c
- Consider Jacques Hadamard's argument
- Open and explore the SSA GeoGebra by t. picone above the video and sketch and label the results in your notes

congruence and similarity 2 interactive video

- Consider what is the smallest amount of information we need to determine that quadrilaterals are congruence or similar?
- Take a look at SSSS quadrilaterals above the video. Explore the GeoGebra by sliding and then by dragging the one vertex. In your notes, sketch pictures and annotate what is the implication of the exploration.

- Next, going back to triangles first, look at AA triangles above the video. Explore the GeoGebra by the slide me slowly controller. In your notes, annotate what is the implication of the exploration.
- Returning to quadrilaterals, look at AAA quadrilaterals above the video. Explore the GeoGebra by sliding and then by dragging one of the vertices in the bottom right. In your notes, sketch pictures and annotate what is the implication of the exploration, both for AAA and for AAAA in a quadrilateral.
- Does AA in a Euclidean triangle imply AAA in that triangle? If so explain why and if not, provide a counterexample.
- Does SSSS and AAAA seem to give congruence or, if not, similarity? If not, provide counterexamples.
- Consider quadrilaterals $DABC$ and $EABC$ as related to SSASA in this diagram. What is congruent and what is not congruent?
- Sketch the counterexample in your notes and label the relevant sides and angles.
- List what else are these a counterexample for in your notes.
- Next, consider ASSA similarity for a quadrilateral.
- Sketch $ADCB$ and $A'DCB'$ in your notes and consider ASSA for them.
- How many angles or sides can suffice to determine congruency or similarity in a quadrilateral?
- Given the units of volume, what might we expect volume to be proportional to?
- Consider the Excel steps so that you can do them in a later project, try them yourself if you have access to Excel now, and consider the strengths and weaknesses of the model. You can obtain a working copy of Microsoft Excel, if you like, which is free to ASU students as on `https://support.appstate.edu/services/catalog/microsoft-office`.
- Consider the two models for the bass fish. Which is better correlated to weight?

Euclidean and spherical perspectives interactive video

- What is intrinsically straight on a sphere?
- We can consider $41.885°$ N, $87.63°$ W in the area of Chicago, Illinois, USA and $41.885°$ N, $12.495°$ E in the area of Rome, Italy. They are on the same latitude. Is the non-equator latitude between Chicago and Rome intrinsically straight on the sphere?
- Read through Euclid's 5th postulate, Postulate 5, as well as Proposition 31.
- How many intrinsically straight great circle parallels can we find to a given great circle through a point off of it?
- Open Walter Fendt's App which is a link above the video and drag the vertices of the spherical triangle made from portions of 3 great circles. Consider how big the angle sum (the bottom right number) and how small the angle sum can get and what kinds of configurations lead to these. Sketch some resulting spherical triangles in your notes and label the sum of the angles on them.
- What is the largest angle sum of a spherical triangle?
- How small can the angle sum of a spherical triangle get for 3 points that are not all on the same great circle?

- Use the Pythagorean theorem to test whether 20, 21, 29 form a right Euclidean triangle or not.
- What would happen to the Pythagorean theorem on the sphere for large triangles?
- What would happen to the Pythagorean theorem on the sphere for small triangles?
- Can we form squares on a sphere consisting of four right angles (90 degrees) and four equal sides that are portions of intrinsically straight great circles?
- For the dodecahedron (shown on the slide), how many pentagons come together at each vertex?
- Are there more, less, or the same number of spherical polyhedra compared to Euclidean polyhedra?
- Sketch the 2 triangles, the one as the 2 marked sides and the dotted connection, a typical spherical triangle, and the other that includes the hemisphere below too, with the last side the long way around the great circle (but still an intrinsically straight path). Sketch these in your notes.
- Write the argument of why AA implies AAA in Euclidean geometry.
- Sketch the 2 spherical triangles ABC and AB'C' in your notes, with the intersecting triangle shaded.
- Sketch the 2 spherical lunar triangles ABC and AB'C' in your notes.
- Write down the formula for the surface area of a sphere as a function of the radius r.
- For area of a plane and surface area of a sphere, in terms of the metric system, what are the units expressible as?

Pythagorean theorem interactive video

- What is a side length of the small square?
- Foil and reduce.
- Next sketch the picture and write the steps and reasons in your notes.
- Read through Proposition 13 in Euclid's Elements, write it in your notes, and consider how it applies here.
- Label your figure to show this and clarify the sides opposite the 2 angles in the bottom vertex.
- Can you see why this gives us a right angle at that vertex?
- Write out the argument that if we start with a triangle we can form the figure from the 周髀算經 or Zhoubi Suanjing using transformations and show that we do have squares.
- Watch the water wheel Pythagorean theorem video from the link above this video.
- Why are the triangles congruent?
- Write Proposition 41 from Euclid's Elements in your notes and consider how it applies here.
- What goes wrong in spherical geometry?
- Watch the mathematicsonline video of Proposition 47 from the link about the video and take notes on aspects you like and aspects that could be improved.
- In your notes, write down items related to the strengths and differences of the two proofs of Proposition 47.

analytic geometry and metric perspectives interactive video

- In your notes, write down two points and calculate their distance by hand by using the Pythagorean metric.
- For the coordinate geometry of the plane, the first coordinate tells us how far to the right on the positive horizontal x-axis to go. The second coordinate tells us how far up the positive vertical y-axis to go. (A negative would be in the opposite direction). Set axes and units, graph $A = (2,5), B = (2,1)$, and $C = (4,1)$ and label the vertices and their coordinates.
- Graph $A' = (5,4)$, $B' = (3,2)$, and $C' = (4,1)$ and label their vertices and coordinates.
- What is the taxicab distance between $A' = (5,4)$ and $B' = (3,2)$?
- Compute the taxicab distances from $A = (2,5)$ to $C = (4,1)$ and separately from $A' = (5,4) to C' = (4,1)$.
- Write a proof in your notes.
- Review the Euclidean proof of SAS (I-4) from the congruence and similarity 1 interactive video notes you took.
- Where does the proof of SAS (I-4) first fail in taxicab geometry?
- Open the taxicab tool from the link above the video
- Compute the Pythagorean distance when the horizontal distance is 490 and the vertical distance is 764. approximately 908 feet
- In your notes, sketch the taxicab squares, label their taxicab lengths in terms of the number of blocks traveled, and the areas 9 blocks^2 versus 8 blocks^2
- Notice that we are approximating a curved region with a flat one on an infinitesimal level
- Consider how the Euclidean metric and algebra came in to the derivation of the surface area of a surface of revolution
- Given $f(x) = \sqrt{r^2 - x^2}$, compute $f'(x)$ using chain rule
- Find a common denominator of the second square root and reduce
- Include the integral computations and write this as a proof that the surface area of a sphere is 4 pi times the radius squared. Notice that we relied heavily on analytic geometry and metric perspectives. We'll discuss later how Archimedes and others were able to approach problems like this long before coordinate geometry!
- If we cut a spherical loaf of bread into equal width slices, which has the most crust?
- Divide the surface area of a piece cut like before by the surface area of the entire sphere. What do we obtain as a function of the width of the piece and the radius of the sphere?
- Sketch the triangle and compute the $\sin(23.5°)$ in your notes. In addition, approximately what percentage of the earth is between the tropics?

polyhedra and angle defect interactive video

- What is $V - E + F$ of a region in the plane that satisfies the rules in the experiment?

- In your notes, write down $V - E + F$ and 120-degree axis for each regular polyhedra.
- What is p, k and n for a dodecahedron?
- How can we express V in terms of n, k, and perhaps p?
- Write down the proof of the inequality in your notes.
- When $p = 3$ and $k = 6$ what happens to $2/p + 2/k$?
- Write the rest of the argument that there are 5 regular Euclidean polyhedra in your notes.
- Why does $k = 2$ work on a sphere but not the plane?
- What is the interior angle of a hexagon?

measurements and angle sum interactive video

- Write Archimedes' argument for the area of the circle in your notes.
- Approximately what fraction of the cylinder does the sphere take up?
- What fraction of the cylinder does the cone take up?
- Sketch and label the cylinder, sphere, and cone.
- Use known formulas to show the volume of this cylinder = volume sphere + volume cone.
- What familiar theorems are assumed?
- Sketch in your notes a visualization that shows how to walk a Euclidean triangle angle sum of 180 degrees.
- What can we logically infer from this relationship (and write your reasoning in your notes)?
- What is the sum of the angles minus π for a triangle that goes from the North Pole to the equator, one-quarter the way around, and back up?
- Sketch a pic and what axiom lets us construct the parallel?
- Write this part of the proof of I-32.
- Write the rest of the proof of I-32.
- What first goes wrong with the proof on the sphere?
- Write the computation in your notes.
- What is the angle sum in this triangle in M.C. Escher's model?

parallels 1 interactive video

- Write down as many definitions or connections to parallels as you can think of.
- What are intrinsically straight paths in spherical geometry?
- What are intrinsically straight paths in Escher's hyperbolic model?

- How many intrinsically straight parallels to a line through a point off it are there in Escher's hyperbolic model?
- Write down the negation.
- How many ways can we consistently fill in this game?
- Where is Euclid's 5th true?
- Where is Playfair's true?
- Think about why I-12 first and then I-11.
- Think about what is wrong and which proposition it contradicts.
- Write the proof of I-31, the existence portion of Playfair's.
- What first goes wrong with the proof of I-31 on the sphere?
- How many intrinsically straight parallels to a line through a point off it are there in spherical geometry?
- Sketch a picture as you write the details related to I-16.
- Label the SAS in the 2 triangles in your picture.
- Sketch a picture of a spherical triangle and E coming back down and meeting the extension of AB so that the exterior angle is not larger.

parallels 2 interactive video

- Write the statements of Playfair's and Euclid's 5th in your notes. Select "true" when ready.
- Sketch a picture for the proof.
- Which proposition is this?
- Highlight the 4 angles in your picture.
- Which propositions were needed to prove Playfairs + prior to I-28 $\rightarrow$ Euclid's 5th?.
- Review the proof of I-31 from notes you previously took for parallels 1 and write the propositions we used.
- Why aren't Euclid's 5th and Playfair's logically equivalent? Consider a geometry where one is true and the other is false.
- Review your responses on the worksheet on hyperbolic geometry 1 for Hyperbolic Shortest Distance Paths.
- Review your responses for Hyperbolic Angle Sum.
- Review your responses for Hyperbolic Euclid's 5th.
- Review your responses for Hyperbolic Playfair's.
- Review your responses for SMSG 16.
- Consider these pictures.

- Fill in each entry as true or false in each geometry.

parallels 3 interactive video

- Which lets us construct these perpendiculars?
- Sketch a picture and label it.
- Which lets us construct the segment?
- Which proposition shows the angles are congruent?
- Which proposition shows the triangles are congruent?
- What goes wrong in the non-Euclidean geometries?
- Which is true for shortest hyperbolic paths?
- Which is true for the existence portion of Playfair's proof?
- Is the existence portion of Playfair's true in spherical geometry?
- Is the existence portion of Playfair's in hyperbolic geometry?
- What would the 'problem' be if the real universe is spherical?
- What geometry did Lobachevsky's experiment indicate?
- What geometry did Kirshner's experiments seem to indicate?
- What geometry did the density experiments indicate?
- How do polyhedra relate to possibilities for the geometry of our universe?

projective geometry interactive video

- Evaluate the arguments. Select "true" when ready.
- Review and sketch the diagram of the existence part of Playfair's. Include P, p, and A. Select "true" when ready.
- Think about what happens to the exterior angle in hyperbolic and spherical geometry? Select "true" when ready.
- What happens to Proclus's assumption in hyperbolic geometry?
- How do the planes intersect?
- What happens to Desargues when corresponding sides of a triangle are parallel?
- What happens to Desargues in hyperbolic geometry?
- What happens to Desargues on the sphere?
- Think about explaining division to a child. Select "true" when ready.
- What is projective geometry?
- Does SAS hold in projective geometry?
- Which do you find most compelling about why Euclid included the 5th postulate?