Worksheet on Axiomatic Systems, Measurement, and Constructions

Dr. Sarah's MAT 3610: Introduction to Geometry

goals:

- IGS Exploration
  I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate they seem to apply in a wide variety of examples.
- Proof Considerations
  I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
  I can compare and contrast multiple geometric perspectives.

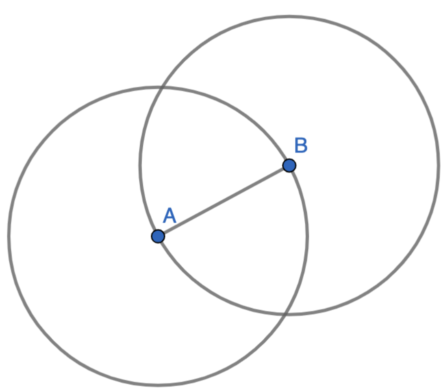
1. Open an Interactive Geometry Software (IGS) and take out Book 1 of *Euclid's Elements* handout.

Equilateral Triangle I-1 and Perpendicular I-11

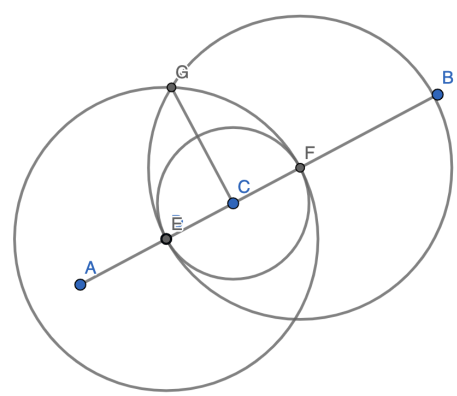
2. Start with a segment. Construct an equilateral triangle with only straightedge, compass, and intersection features.

3. Measure the interior angles and side lengths and then drag the vertices to demonstrate the construction seems to apply in a wide variety of examples.

4. What was the key point in the proof of I-1—in terms of why the triangle we constructed was actually equilateral?



I-1



I-11

5. Which congruence theorem was key in the proof of I-11 to show that we had constructed a perpendicular?

Bisect a Segment: I-10

6. Begin a new document with a segment $\overline{AB}$. Using only straightedge, compass, and intersection features, construct the midpoint (hint: use both intersection points from the I-1 construction).

7. Measure the distances between A and the midpoint and between B and the midpoint. Then drag the vertices to demonstrate the construction seems to apply in a wide variety of examples.

8. View the script/commands of your constructions (in GeoGebra this is the calculator symbol)
9. Use I-1 and I-9 among other reasoning from Euclid's Book I in order to write a rigorous proof in paragraph form which identifies underlying assumptions and justifies I-10. You may refer to Euclid's definitions, postulates, common notions up through I-9.
10. When you have finished your proof, compare with a neighbor and/or show it to me.
11. Compare and contrast the construction in the IGS and the proof.

Perpendicular Bisectors of a Triangle

12. Begin a new document with a triangle ABC. Use the built in features of the IGS to construct the three perpendicular bisectors of $\overline{AB}, \overline{AC}$, and $\overline{BC}$.
13. Notice that the three perpendicular bisectors seem to meet. Drag the vertices of the triangle to explore where/when they meet and find a relationship between where the bisectors intersect and a geometric measurement of the triangle. What kind of triangles, if any, have the perpendicular bisectors meeting inside the triangle? Outside the triangle? On one of the sides of the triangle?
14. Next, construct a circle with center at the intersection of the bisectors and radius at one of the vertices of the triangle. Drag one of the vertices as you notice that A, B, and C all seem to lie on the circle. The circle is called the circumcircle and the center is called the circumcenter.
15. Go to the calendar webpage and read through an argument of why the perpendicular bisectors intersect (under today's date) and take notes on it. What congruence theorem is key?

Wile E Coyote Axiom System

16. I keep having a recurring nightmare where I am trapped in the following axiom system:

    - A1: Coyotes and roadrunners live on the surface of a perfectly round planet.
    - A2: Coyotes only begin chasing roadrunners exactly 2 seconds after the roadrunner passes them.
    - A3: Coyotes can only catch roadrunners if they can catch up to them after having chased them.
    - A4: Roadrunners run faster than coyotes.
    - A5: Coyotes stop chasing roadrunners when they disappear from view.
    - A6: All coyotes have 20/20 vision.

    Will I be able to catch the roadrunner? If needed, can you add other axioms to the system, which are consistent with A1 through A6, that will ensure that I will always catch the roadrunner? Help me—you're my only hope!

    Hungry as ever,

    Wile E. Coyote