

Worksheet on Desargues' Theorem 1

Dr. Sarah's MAT 3610: Introduction to Geometry

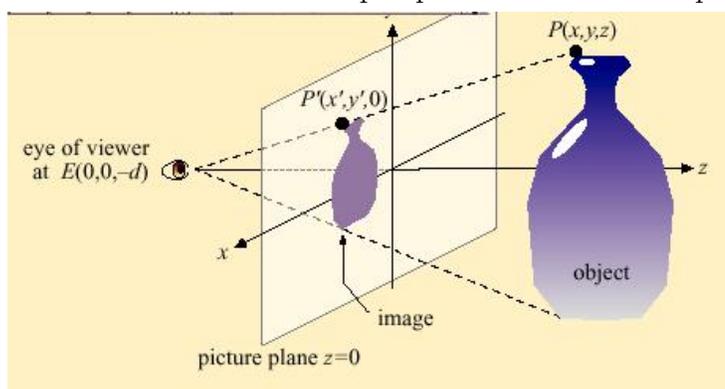
goals:

- IGS Exploration
I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate they seem to apply in a wide variety of examples.
- Proof Considerations
I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
I can compare and contrast multiple geometric perspectives.

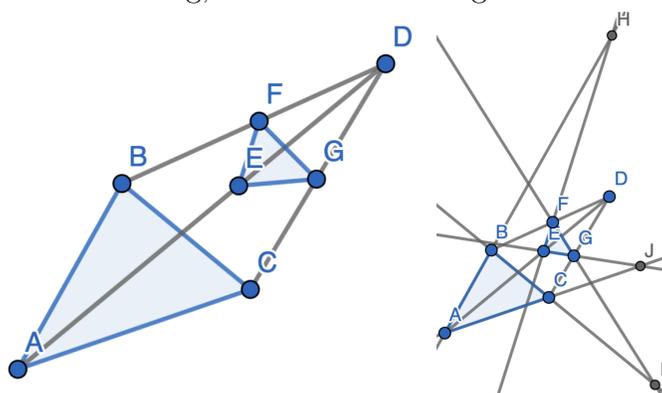
Welcoming Environment: Keep it a safe place to express meaningful ideas and opinions. Actively listen to others and encourage everyone to participate. Part of the welcoming environment is to keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

1. Desargues' Theorem, named for French mathematician Girard Desargues, is a famous result in projective geometry: when two triangles are in perspective then the 3 intersections of the corresponding sides are collinear. We will first explore this statement in Euclidean geometries. Open up GeoGebra Geometry.

- Use the **Polygon** tool to create triangle ABC . You will have to click back on A to complete the triangle.
- Create a **Point** D outside the triangle and line **Segments** \overline{AD} , \overline{BD} , and \overline{CD} .
- Use the **Polygon** tool to create triangle EFG with E on the segment \overline{AD} , F on the segment \overline{BD} , and G on the segment \overline{CD} (see the image on the right below).
- **Move** E to be sure it is constrained to the segment \overline{AD} , and similar for F and segment \overline{BD} , and G and segment \overline{CD} . Notice that two triangles are in perspective with respect to the point D . The term "in perspective" arises from perspective drawing, like in the left image below.



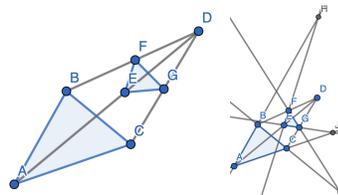
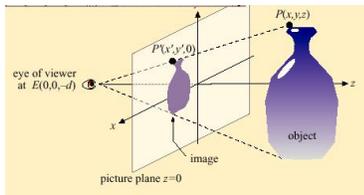
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- Next, create the **Lines** \overline{AB} and \overline{EF} .
- If needed, Zoom out or move some points, so that you can see their intersection (if they are parallel, move one of the points so they are not). Then **Intersect** the lines \overline{AB} and \overline{EF} at point H .
- Similarly, create the intersection of lines \overline{BC} and \overline{FG} , point I , and the intersection of lines \overline{AC} and \overline{EG} at point J .

Do the 3 intersections of the corresponding sides seem to be collinear as in Desargues' theorem? (if not, try again)

2. What happens if we move the point D . What else changes and what remains the same?
3. What happens if you move the point E ? What else changes and what remains the same?
4. What happens to Desargues' theorem if one or more sides of the triangle EFG are parallel to the sides of triangle ABC ?
5. To show that Desargues' theorem holds, imagine lifting D and triangle EFG into 3-space, kind of like the picture with the vase.



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Now if ABC and EFG lifted are in two different planes that are not parallel, then what is the intersection of those two planes?

6. How would this show that H , I , and J must be on the same line once we project back to 2-space?
7. Consider Desargues' theorem in non-Euclidean geometries and discuss with your group.

8. **Help each other and PDF responses to ASULearn:** If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. If your entire group is finished, then split up and pull up chairs so that you can discuss your responses with other groups. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.