

Worksheet on Hyperbolic Geometry Part I

Dr. Sarah's MAT 3610: Introduction to Geometry

goals:

- IGS Exploration

I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate they seem to apply in a wide variety of examples.

- Geometric Perspectives

I can compare and contrast multiple geometric perspectives.

Welcoming Environment: Keep it a safe place to express meaningful ideas and opinions. Actively listen to others and encourage everyone to participate. Part of the welcoming environment is to keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

1. **Building Community:** What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

Shortest Distance Paths

2. Open up <https://www.geogebra.org/m/d845j9a3>. We have two paths between A and B . Notice that the path through C looks like the intrinsically straight paths that cut angels and demons in half in Escher's *Circle Limit IV: Heaven and Hell* that we looked at in the measurements and angle sum interactive video. Sketch a picture of the two paths.
3. Which path is shorter using the hyperbolic metric in this model? The path from A to B through C or through D ?
4. Drag B up toward A (but keep the configuration the same with C to the left of D and B below them). What happens to the difference between the distances?
5. Open up <https://www.geogebra.org/m/R5e9AggU>. Under the first wrench tool, find the **Hyperbolic Segment** tool. Create a hyperbolic segment. Then select the (usual) move symbol (pointer) and drag the endpoints around but inside the disc. Can you obtain the different types of paths that Escher represented as cutting angels and demons in half? Sketch pictures here:

Sum of the Angles

6. What was the sum of the angles at the three points we explored in Escher's *Circle Limit IV: Heaven and Hell* in the measurements and angle sum interactive video?
7. How did we compute the sum of the angles there?

8. Open up <https://www.geogebra.org/m/svywsx3r> to explore the sum of the angles in hyperbolic geometry more generally. Drag A , B , and C but keep the same configuration (so that you don't end up with an exterior angle measurement—i.e. don't let A cross \overline{BC} or its extension, and similar for the other points) and keep them all inside the disc. How small can the sum of the angles get in this IGS exploration? What kind of triangle must we form in this model to get a small angle sum?
9. How large can the sum of the angles get in this IGS? What kind of triangle must we form in this model to get a large angle sum?
10. Sketch pictures that illustrate your prior two responses.

Euclid's 5th Postulate

11. Write down the statement of the 5th postulate from *Euclid's Elements* Book I
12. Open <https://www.geogebra.org/m/qmbmeas9>. Do the intrinsically straight paths in the sketch seem to satisfy the assumptions/conditions as well as the conclusion of Euclid's 5th Postulate?
13. Sketch a picture and identify the components in Euclid's 5th.
14. Drag point E so that the assumptions/conditions of Euclid's 5th Postulate still show as holding. Does the conclusion always appear to continue to hold?
15. If not, sketch a picture and identify the components in the contradiction of Euclid's 5th.
16. Based on #12–15, is Euclid's 5th postulate true in hyperbolic geometry? A postulate holds in a geometry if it holds in all cases.
17. **Help each other and PDF responses to ASULearn:** If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. If your entire group is finished, then split up and pull up chairs so that you can discuss your responses with other groups. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.