

Worksheet on Hyperbolic Geometry Part 3

Dr. Sarah's MAT 3610: Introduction to Geometry

goals:

- IGS Exploration
I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate they seem to apply in a wide variety of examples.
- Proof Considerations
I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives
I can compare and contrast multiple geometric perspectives.

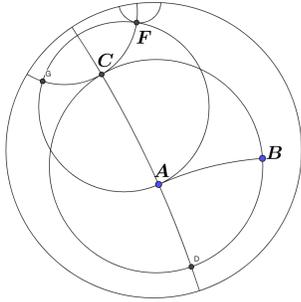
Welcoming Environment: Keep it a safe place to express meaningful ideas and opinions. Actively listen to others and encourage everyone to participate. Part of the welcoming environment is to keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

1. **Building Community:** What are the preferred first names of those sitting near you? If you weren't able to be there write N/A or give reference to anyone you had help from.

Pythagorean theorem

2. Open up
<https://www.geogebra.org/m/jxejcdy5>
to explore the Pythagorean theorem in hyperbolic geometry. Does the Pythagorean theorem appear to hold in this model of hyperbolic geometry?
3. Drag the vertices but keep them all inside the disc and keep the configuration the same. What seems to happen to the Pythagorean theorem in small triangles?
4. Sketch pictures that relate and identify geometric components.
5. Open up
<https://www.geogebra.org/m/R5e9AggU>
 - under the first wrench tool, find the **Hyperbolic Segment** tool and create a segment \overline{AB}
 - under the second wrench, apply the **Hyperbolic Perpendicular At Point** by selecting A first and then B . You should now have a hyperbolic perpendicular through A to \overline{AB} .
 - under the first wrench, use the **Hyperbolic Circle** to select A first and then B . This is a hyperbolic circle with center A and radius \overline{AB} .
 - under the point tool, choose **Intersect** and select the perpendicular and the circle.
 - next apply the **Hyperbolic Perpendicular At Point** again, but now select C first and then A .

- apply the **Hyperbolic Circle** and choose C and then A
- Intersect** the new circle with the latest perpendicular.
- Select the intersection point closest to B and use **Hyperbolic Perpendicular At Point** to select that point (F in my diagram) and C .



We have three equal width segments (\overline{AB} , \overline{AC} , and \overline{CF} in my picture) and three 90° angles. In Euclidean geometry, the figure would have already closed up to form a square. Here in this model does this form a square that encloses four 90° angles and four equal sides?

6. Drag A and B —can we form a square as we move A and B around?
7. As a review, sketch a diagram from the Euclidean proof of the Pythagorean theorem from *Euclid's Elements*.
8. As a review, sketch a diagram from the 周髀算經 or *Zhoubi Suanjing* proof and puzzle of the Euclidean Pythagorean theorem.
9. Based on your explorations above, what goes wrong with these Euclidean proofs in hyperbolic geometry?
10. **Help each other and PDF responses to ASULearn:** If you are finished with the worksheet before I bring us back together, first ensure that your entire group is finished too, and if not, help each other. If your entire group is finished, then split up and pull up chairs so that you can discuss your responses with other groups. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment. I recommend you turn it in sometime today, but you have until the next class.