

# Worksheet on Congruence and Similarity

## Dr. Sarah's MAT 3610: Introduction to Geometry

goals:

- IGS Exploration  
I can use Interactive Geometry Software (IGS) to discover relationships and demonstrate they seem to apply in a wide variety of examples.
- Proof Considerations  
I can write rigorous proofs in geometry, identify underlying assumptions, and understand limitations and applications.
- Geometric Perspectives  
I can compare and contrast multiple geometric perspectives.

### Euclidean SSA

1. Explore <https://www.geogebra.org/m/E8cZQwjV> to visualize two different triangles with the same SSA that are not congruent.
2. With the measurements listed there, use the law of cosines in order to solve for the third side of the triangle (with help from the quadratic formula).
3. Use your last response to help you write an analytic/metric geometry proof in paragraph form that two triangles satisfying SSA do not necessarily have to be congruent (assume that the law of cosines holds and provides valid Euclidean triangles).
4. Skim through [https://artofproblemsolving.com/wiki/index.php/Law\\_of\\_Cosines](https://artofproblemsolving.com/wiki/index.php/Law_of_Cosines). What theorem is needed to prove the law of cosines?

### Transformations

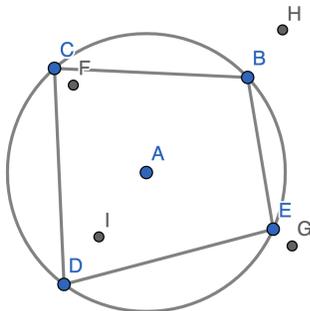
5. Open <https://www.geogebra.org/geometry/dpu4xqhw>
6. Use the following Transform Tools to get used to these as you explore the effects of transformations on congruence and similarity of the image.
  - Translate by Vector
  - Rotate around Point
  - Reflect about Line
  - Dilate from Point

### Shearing a Euclidean Rectangle

7. Go to <https://www.geogebra.org/m/pDU4peV5> and slide  $b$  in order to see the effect of a horizontal shear.
8. Is similarity preserved?
9. If not, can you find geometric measurements that are preserved?

## Euclidean Quadrilateral Orthocenters

10. Start a new IGS document.
11. Construct a circle with center  $A$  and radius  $AB$ , i.e.  $B$  is a point on the circle.
12. Construct 3 additional points on the circle,  $C, D, E$  as in the configuration below so that  $C$  is somewhere in between  $B$  and  $D$ , and  $D$  is in between  $C$  and  $E$ , and you don't form  $90^\circ$  angles.
13. Connect the edges to form a quadrilateral  $BCDE$ .
14. If there are any right angles in  $BCDE$  then drag the points to ensure the angles are NOT  $90^\circ$ .
15. Construct the diagonal  $\overline{BD}$  so that you have  $\triangle BCD$ .
16. Construct  $F$  as the orthocenter of  $\triangle BCD$  (the intersection of the altitudes of the triangle—an altitude is perpendicular to a base through a vertex of a triangle). So use the Perpendicular Line tool to select a vertex and its opposite side. Once you have the three altitudes, use the intersect tool to construct  $F$ .  
After you are done, hide the perpendiculars you used to create the orthocenter, but don't hide  $F$ ! (you can use the Show/Hide Objects tool under Edit or select one or more objects, then the three dots, then Settings and then unselect Show Object)
17. Similarly, construct  $G$  as the orthocenter of  $\triangle BED$ .  
Next hide the perpendiculars and the diagonal  $\overline{BD}$ .
18. Construct the diagonal  $\overline{CE}$  so that you have  $\triangle CBE$ .  $H$  as the orthocenter of  $\triangle CBE$ , and  $I$  as the orthocenter of  $\triangle CDE$ .  
After you are done, hide the perpendiculars you used to create the orthocenters and the diagonal  $\overline{BD}$  but don't hide the orthocenters themselves.
19. Construct the quadrilateral  $FHGI$  and compare it with  $BCDE$ . What do you notice?
20. Drag  $D$  to see whether the relationships seem to hold in a wide variety of examples.
21. Next drag  $A$  to do the same.



## Euclidean SSSAA

22. We know that SSA does not provide similarity in triangles (see 1.). As an extension, construct two quadrilaterals that satisfy SSSAA but are not congruent.