

Project 2: Euclidean & Spherical Geometry Applied to Course Topics

You may work alone or with one other person. During class you will write down your top problems (unranked) and from this you will be assigned one problem. In this project, we will use intuition from previous Euclidean geometry knowledge along with experiences from hands-on models and/or research. The purpose of this assignment is an introduction to the concepts in the course syllabus as we explore diverse geometric perspectives.

Project Components:

- a) **Review content related to your problem that applies to the geometry of the plane / Euclidean geometry / flat geometry.** For example, if you have Problem 1 then you might review the definitions of lines and how to calculate them, for Problem 9 you might review some information related to area and surface area, for Problem 10 you might review coordinate systems... You may use our book, the internet, or other sources.
- b) **Why the topic is important in mathematics and the real-world?** Conduct searches like “triangles are important” or a search word related to your problem and then (with and without quotations) words like important, interesting, useful, or real-life applications. You might also pick some specific fields to see if there are applications such as: Pythagorean theorem chemistry
- c) **Summarize diverse spherical perspectives.** You do not need to prove an answer or completely resolve the issue on the sphere. You should look for various perspectives related to spherical geometry, and summarize those in your own words. Try different combinations of search terms related to your problem along with words like sphere, spherical, earth, or spherical geometry. Vary your word combinations: spherical Polyhedron versus Polyhedra on a sphere yield very different results. Quotations can be helpful if there are too many results: “straight lines on a sphere.” In the book you can search in the appendix for double elliptic, sphere, and spherical geometry, which will yield different pages related to some of these problems.
- d) **References.** Keep track of your sources and list them in a consistent format.

As you respond to a), b) and c) summarize the ideas in your own words. Your grade will be based on the diverse perspectives you provide and the clarity, depth, and professionalism of your work. In addition, be prepared to share what you found with your classmates during class, so you may want to keep a copy of your work.

We will each select one problem during class:

1. Intrinsically Straight

A straight line on the surface of a sphere must curve from an extrinsic or external viewpoint, but intrinsically, say for example if we are living in Kansas, we can define what it means to feel like we are walking on a straight path. What is straight (intrinsically) on a sphere? Is the equator an intrinsically straight path? Is the non-equator latitude between Chicago and Rome an intrinsically straight path?

2. Parallelism

Given a spherical path l and a point P off the path, how many parallels are through P to l on the surface of the sphere?

3. Triangle Angle Sum

What is the sum of the angles in a spherical triangle?

4. Pythagorean theorem

Assume that we have a right-angled spherical triangular plot of land (a curved triangle formed by three shortest distance paths on the surface of the sphere that also contains a 90 degree angle) on the surface of a spherical globe between approximately the north pole, a point on the equator, and a point one-quarter away around the equator. Do the sides satisfy the Pythagorean theorem?

5. Squares

On the surface of a perfectly round beach ball representing the earth, if we head 30 miles West, then 30 miles North, then 30 miles East, and then 30 miles South, could we end up back where we started? Must we? What about 3000 miles in each direction? Can we construct a square on a sphere?

6. Polyhedra

Can we construct every convex polyhedron on a sphere, like a soccer ball (a spherical version of a truncated icosahedron)? Are there spherical polyhedra that have no flat equivalents?

7. Congruence

Is SAS (side-angle-side) always true for spherical triangles on the surface of a perfectly round beach ball?

8. Similarity

Similar triangles satisfy AAA (angle-angle-angle). Can we construct similar triangles on a sphere?

9. Surface Area

If we slice a perfectly round loaf of bread into equal width slices, where width is defined from the center using a straight edge or ruler, which piece has the most crust (or surface area)? Why?

10. Coordinate Systems and Dimensions

What coordinate systems apply to the surface of a sphere? Is the surface of a sphere 2-D or 3-D?

I have texts in my office that may be helpful in addition to our course book and web searching you do.