Two Different Approaches to the Following consistent Minesweeper game:

	A	B	C	D	E	F	G
1		0	0	0			
2			2				0
3	*	*					1
4	2	3		1			1

Theorem 1: A3 = * = B3

Assume that we have already proven Theorem 1. There are lots of possibilities for what we can do next. Here are two different approaches and their proofs.

Approach 1

Theorem 2: Exactly 1 of C3 and D3 is a bomb while the other is a #

Theorem 3: C4 = 2

Notice that in Approach 1, we can determine C4 even though adjacents C3 and D3 have not been determined. This is similar to the approach needed in project 2.

Approach 2

Theorem 2: C3 = *

	A	B	C	D	E	F	G
1		0	0	0			
2			2				0
3	*	*					1
4	2	3		1			1

Proof of Approach 1

Theorem 2: Exactly 1 of C3 and D3 is a bomb while the other is a #

Theorem 3: C4 = 2

Proof of Theorem 2: We will show that exactly 1 of C3 and D3 is a bomb while the other is a #. Look at C2=2 which has one adjacent * in B3 by Theorem 1. By axiom 2, it must have another adjacent *. By proposition 1, B2=# since B1=0 and B2 is adjacent to B1. Similarly, D2=#. Since the only remaining squares adjacent to C2=2 are C3 and D3, exactly 1 of them must be a bomb. Then, by axiom 1, the other must be a #. Hence, we have shown that exactly 1 of C3 and D3 is a bomb while the other is a #, as desired.

Proof of Theorem 3: We will prove that C4=2. First notice that by axiom 2, C4 is not a * as D4=1 already has its adjacent * in either C3 or D3, by Theorem 2. Hence, by axiom 1, C4 is a #. We will examine the squares adjacent to C4. We have already proven that B3=* in Theorem 1 and that exactly one of C3 and D3 is a bomb while the other is a # in Theorem 2. Since the other squares adjacent to C4 are #s, then C4 has exactly 2 adjacent *s (one in B3 and the other in either C3 or D3). Hence, by axiom 2, C4 is a 2, as desired.

	A	B	C	D	E	F	G
1		0	0	0			
2			2				0
3	*	*					1
4	2	3		1			1

Proof of Approach 2

Theorem 2: C3 = *

Proof of Theorem 2: In order to show that C3 = *, assume for contradiction that C3=#. Now B4=3 has 2 adjacent *s in A3 and B3, by Theorem 1, and so it must have exactly one more adjacent * by axiom 2. Since the only available squares for this * are in C3 and C4 and we have already assumed that C3=# then we must have that C4=*. Notice that D4=1 now has an adjacent * in C4, and so by axiom 2, all of D4's adjacent squares cannot be *s. Hence, by axiom 1, they must be #s. Namely, D3=#. But, look at C2=2 which has one adjacent * in B3 by Theorem 1. By axiom 2, it must have another adjacent *. Yet, we have assumed that C3=# and we have just shown that D3=#. Also, by proposition 1, B2=# since B1=0 and B2 is adjacent to B1. Similarly, D2=#. We have arrived at a contradiction to the fact that C2=2 since it must have another adjacent *, but all of the other squares are #s. Therefore our original assumption that C3=# must be incorrect since we know that the game is consistent. Thus C3=* by axiom 1, as desired.