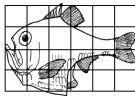
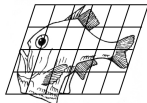


MAT 4040 Project: Transformations

Sarah J Greenwald



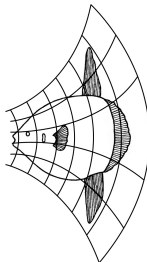
Argyroplecus



Sternoptyx



Diodon



Orthagoriscus

A D'Arcy Thompson Transformation Grid [12]

- Connects geometry, algebra, linear algebra and analysis
- Applications in statistics, physics, computer science, architecture, art, astronomy and optics

Experience with Transformations

Definition

In general a transformation changes some aspect while at the same time preserving some type of structure.

- a) Representations using coordinates, vectors, functions, matrices and groups
- b) Matrix transformations in 2-D and 3-D in MAT 2240
- c) Symmetry groups in MAT 3110
- d) Symmetries of polyhedra in MAT 3610

Criteria Reminder

- Definitions, examples, important results, proofs... (from prior experiences)

Example

$$A = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

represents a counterclockwise rotation by θ in the $x - y$ plane with the z coordinate fixed.

- Historical and recent progress

Rich Early History of Transformations

- Aesthetically pleasing design patterns
- Aristotle's objections
- Euclid of Alexandria: rotations
- Archimedes of Syracuse: axial motions
- Appolonius of Perga: inversion
- Marcus Vitruvius: projections
- Omer Khayyam's objections
- Thabit ibn Qurra and his grandson Ibhrahim ibn Sinan: affine transformations of the plane

A Few Generalizations Related to Classes

- **Sir Isaac Newton**: transformations of coordinate systems
- **August Mobius**: homogeneous coordinates
- **Carl Friedrich Gauss**: linked to linear algebra
- **Gotthold Eisenstein** and **Charles Hermits**: addition and multiplication of linear transformations
- **Arthur Cayley**: composition of transformations

Exs of 19th Century Work and Apps in the 20th Century

- **Sophus Lie**: continuous transformation groups and differential equation [Lie groups in physics]
- **James Maxwell**: equations were invariant under Lorentz transformations [geometry of space-time and beginning of relativity theory]

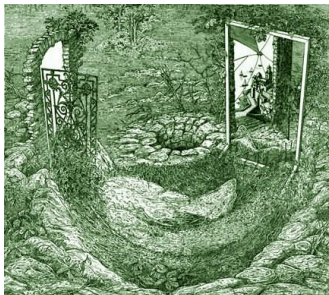
Beginning of the 20th Century:



Felix Klein [11]

- Felix Klein's *Erlanger Programm*
- Transformation group to understand a space
- Link with algebra revolutionized geometry

One Application at the End of the 20th Century



The Well by Istvn Orosz (1998)

Translations, rotations and reflections do not distort, but dilations and inversions do. Invert the image in a cylindrical mirror in the well.



Cylindrical inversion of Istvn Orosz's *The Well* [9]



Cylindrical inversion of Istvn Orosz's *The Well* [9]

M.C. Escher, who was known for transformations

- Escher remarked: *I express transformations in reality. I often do this by having a mirror in my picture reflect a scene that does not fit at all. Sometimes I make subtle changes to my picture that by the other side makes it completely different, a flock of birds changes into a city scape.* [Metropolitan Mathematician's Journal, 1948]
- Challenge by [Bruno Ernst](#)