

Cone Geodesics by Symmetry and Covering Arguments

Dr. Sarah's Differential Geometry

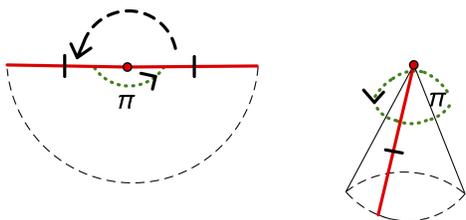
Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as well as when I bring us back together.

- Consider symmetry arguments on the physical cone. Is a latitude (horizontal circle) on a cone a geodesic? Discuss and then respond on pollev.com/drsarah314
 - yes and I have a good reason why
 - yes but I am unsure of why
 - no but I am unsure of why not
 - no and I have a good reason why not
- Next, use string pulled tightly to find geodesic paths between 2 points and consider symmetry arguments as you discuss: Is there always at least one geodesic between any two points? Can a geodesic ever intersect itself? What are the different kinds of geodesics on the surface of the cone? In your notes, sketch pictures and take notes.
- What happens when a bug gets to the cone point along a vertical geodesic? Discuss and then respond on pollev.com/drsarah314 after I've cleared the first set of responses
 - the geodesic ends there
 - the bug can continue to walk straight through the cone point to the "other side" by bisecting the cone angle there
 - other

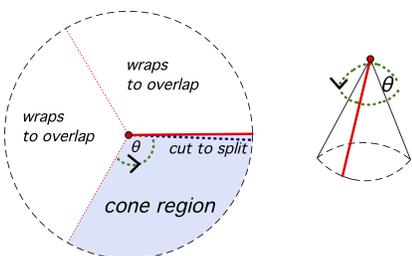
For each question below, sketch pictures in your notes and/or annotate the models.

- A cone can be thought of as a generator from a cone point that sweeps out the cone angle, as with the red generator in this π cone:



Notice we can wrap twice from the two unique sheets in the 2-sheeted covering. Straight lines in the covering map to geodesics in the cone so how many geodesics typically join two points on a π cone? Why?

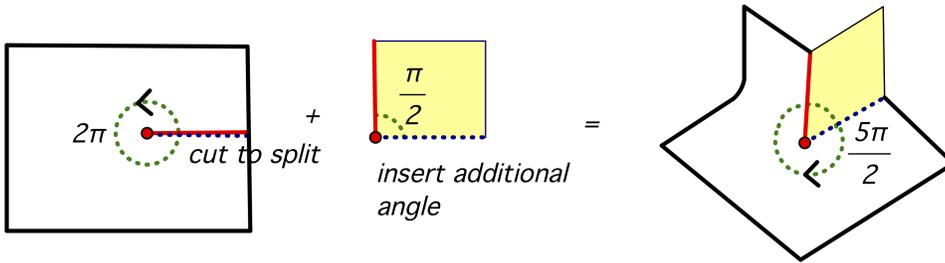
- On a $\frac{2\pi}{3}$ cone, how many geodesics typically join two points? Why?



6. Consider $0 < \text{cone angle} < 2\pi$. In general, on a cone of small enough cone angle, a geodesic
- won't intersect itself
 - will intersect itself a finite number of times with a maximum crossing number that depends on the specific cone angle via roughly the number of times that the cone angle fits into 2π
 - will intersect itself a finite number of times with a maximum crossing number that depends on the specific cone angle via roughly the number of times that the cone angle fits into π
 - will intersect itself infinitely many times
 - other

Discuss and then respond on pollev.com/drsarah314

7. Make a $\frac{5\pi}{2}$ cone by taping in the extra $\frac{\pi}{2}$ piece to a rectangular sheet with red to red and dashed to dashed



This appears commonly in buildings as so-called “outside corners” and is its own covering (no sheets). How many geodesics typically join two points?

8. For the $\frac{5\pi}{2}$ cone, is there always at least one geodesic between any two points? Why or why not?
9. Imagine the $\frac{5\pi}{2}$ cone extends indefinitely in all directions. Can we find a point P that is different than the cone point and a geodesic l not through the cone point such that there is more than 1 geodesic through P that does not intersect l ?
- no so this is similar behavior to the usual parallel postulate for the plane where there is only 1 intrinsically straight path to a line through a give point off the line
 - yes so this is different behavior than the plane's parallel postulate
 - other

Discuss and then respond on pollev.com/drsarah314

10. What happens to geodesics on a cone as the cone angle gets close to 0?
11. What happens to geodesics on a cone as the cone angle gets close to but is less than 2π ?