# Dr. Sarah's Differential Geometry—Welcome!

**Welcoming Environment**: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and review MAT 2130: Calculus with Analytic Geometry III via the questions below as you write your responses in your notes. You are free to use online sources too.

Try to help each other! Talk about them one at at time, as you go along, and keep track of any questions your group has. Feel free to ask me questions during group work time as well as when I bring us back together. I'll also try and monitor the open ended response at
`https://pollev.com/drsarah314`
and you can type questions or comments there if you prefer to contact me that way during class.

1. Sit in a group of 4 (if possible) and introduce yourselves to those sitting near you. What are their preferred first names?
2. Given $\vec{v} = \langle 2, 1, 6\rangle$ and $\vec{w} = \langle 3, 1, -1\rangle$, calculate $\vec{v} - \vec{w}$.
3. Write an equation for the line connecting the points $(2, 1, 6)$ and $(3, 1, -1)$ (same points as the endpoints of $\vec{v}$ and $\vec{w}$).
4. Given $\vec{v} = \langle 2, 1, 6\rangle$ and $\vec{w} = \langle 3, 1, -1\rangle$, calculate $\vec{v} \cdot \vec{w}$ and show the calculation.
5. What is a geometric or physical interpretation when $\vec{v} \cdot \vec{w} = 0$ ?
6. Find a tangent vector to the curve $\alpha(t) = (\cos(t), \sin(t), 5)$ at $t = 2\pi$.
7. Find the speed of the curve $\alpha(t) = (\cos(t), \sin(t), 5)$ at $t = 2\pi$.
8. What is the difference between tangent, velocity, and speed?
9. Find the arc length of the curve $\alpha(t) = (\cos(t), \sin(t), 5)$ from 0 to $2\pi$ and show the calculation.
10. Given $\vec{v} = \langle 2, 1, 6\rangle$ and $\vec{w} = \langle 3, 1, -1\rangle$, calculate $\vec{v} \times \vec{w}$ and show the calculation.
11. What is a geometric or physical interpretation of $\vec{v} \times \vec{w}$?
12. Find a vector that is perpendicular to both $\vec{v} = \langle 2, 1, 6\rangle$ and $\vec{w} = \langle 3, 1, -1\rangle$.
13. Using your last response, write an equation of the plane that both vectors lie in.
14. Find a unit vector pointing in the direction of $\langle 3, 1, -1\rangle$.
15. Is $(1, 1, 5)$ on the curve $\alpha(t) = (\cos(t), \sin(t), 5)$? Why or why not?
16. Let $f(x, y) = 1 + 2x\sqrt{y}$. Find the directional derivative at the point $(3, 4)$ in the direction of $\vec{v} = \langle 4, -3\rangle$.
17. What is a geometric or physical interpretation of $\arccos(\frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|})$?

If your group is finished discussing and responding to the above before I bring us back together, then

- Consider why is a line shorter than any other curve $\alpha(t)$ between 2 points in Euclidean geometry?
- Chat to get to know your classmates better and build community!

# Differential Geometry FAQ and Engagement—Optimize your Success and Understanding!

- Where can I find in-class and out-of-class activities?

  On our ASULearn! See the sections organized by due dates as well as a tentative calendar with in-class listings at the top.

  Look for the completion checkmarks on the far right of the activities, with a solid box as one you self-report and a dashed box is earned for a good faith effort when you access an activity or receive a proficient grade by a deadline .

  Projects are also on ASULearn, but they are not typically paired with a checkmark box.

  The ASULearn components work best from scrolling through the activities themselves on a computer. The calendar and the Moodle mobile app does not always show everything as designed, both for visibility and for due dates.


- How do I contact you outside of class?

  need help from me, your classmates, or tech support? at the top of ASULearn (not e-mail!)

  The Zoom link there is for office hours
  Sunday, Tuesday, Thursday 7-7:45pm (yes, pm!)
  Monday, Wednesday 8:15-9am (yes, am!)

  If you can't make Zoom, select the dropdown item listing only you and I to contact me privately, or the whole class to send a message to everyone! Please use a salutation of Dr. Sarah, my preferred name, in communications with me. I strive to answer individual questions at least once a day, including the weekends, although I may respond within class. I prefer that you use Zoom hours as it is easier to discuss material in person.

- What should I do if I don't understand content or something about the course?

  If it is differential geometry content, come see me in office hours and ask me questions during class.

  If it is material from Calculus With Analytic Geometry III, the prerequisite, or earlier courses, it will be helpful to review to make sure you understand the concepts and know how to do a computational example as we use them in class. This includes
  - acceleration and velocity
  - arc length
  - cross product
  - curvature
  - cylindrical and spherical coordinates
  - derivative of a function of one variable whose range is in $\mathbb{R}^3$, i.e. $(x(t), y(t), z(t))$

- directional derivative
- dot product
- equation of a line in 3-space
- equation of a plane
- fundamental theorem of calculus
- gradient
- magnitude, norm or length of a vector
- multivariable chain rule
- normal vector
- parameterizations of curves and surfaces
- partial derivatives of a multivariable function i.e. $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x, y)$
- speed
- surface area
- tangential and normal components of a vector like acceleration
- tangent line
- tangent plane
- tangent vector
- volume

If it is material from Introduction to Linear Algebra, the co-requisite, it would be helpful to pay special attention as these concepts as they come up in 2240 and in differential geometry, or review them if you have already taken the class:

- addition of matrices
- $a_{ij}$ matrix notation
- basis of a space
- determinant of a matrix
- dimension of a space
- eigenvalue $\lambda$ from $A\vec{x} = \lambda\vec{x}$
- linear combination of vectors
- multiplication of matrices, including $A\vec{v}$ and $AB$
- inverse of a 2x2 matrix
- span of vectors
- symmetric matrix $A^T = A$
- transpose of a matrix $A^T = (a_{ji})$ for $A = (a_{ij})$

If it is an ASULearn activity, I have instructions inside each activity link on ASULearn, at the top. Ask questions inside and outside of class. Access (or re-watch) the course intro interactive video, which explains many components. My course design is intentional and based on best practices from the scholarship of teaching and learning including *Make it Stick: The Science of Successful Learning*. Depending on your prior experiences, it may take some getting used to—I'm here to help you!

# Tentative Calendar

While some items have strict deadlines, there is still flexibility built in and multiple pathways for success—videos have multiple chances to succeed and projects can be completed ahead plus there is a revision opportunity for one of the first three projects and one in-class assessment. Attempt readings and videos for completion and take video notes by the listed date when possible as the material builds on itself. Some days are lighter than others and it will help you to progress on upcoming activities in advance, especially major assignments.

| | Class Monday | Between Classes<br>(by just before 1pm Wed.) | Class Wednesday | Between Classes<br>(by just before 1pm Monday) |
|---|---|---|---|---|
| 1/10–<br>1/12 | review 2130<br>obtain rental book from bookstore | -class intro interactive video -read "Curves"<br>-read 1.1 pp. 1–7<br>-lines and Maple intro interactive video<br>-download or access Maple | curvature<br>osculating circle<br>parabola and line | -read 1.1 pp. 8–14<br>-tractrix interactive video<br>-add ASULearn profile pic<br>-Zoom update & profile pic<br>-get to know posting<br>-read the syllabus |

| | | | | |
|---|---|---|---|---|
| 1/19 | State Holiday | | arc length and speed<br>comparing and contrasting curves | -read 1.2 pp. 14–17<br>-$s$, $T$ and physical attributes interactive video<br>-practice submitting PDF |
| 1/24–1/26 | $s$, T, velocity, speed, acceleration, jerk<br>helix computations | -read 1.3 pp. 17–19<br>-TNB 1 interactive video<br>-choice of curve for Project 1 | TNB<br>curve of Archytas<br>cycloid and spiral | -read "How Flies Fly"<br>-read 1.3 pp. 19–20<br>-TNB 2 interactive video |
| 1/31–2/2 | TNB<br>spherical epitrochoid<br>matching activity | -read 1.3 21–25<br>-curvature and torsion implications 1 interactive video<br>-re-engage matching | curvature and torsion<br>Darboux vector<br>fundamental theorem of space curves | -Project 1: research, investigate and present a curve |
| 2/7–2/9 | Project 1 presentations | -read 1.5 pp. 34–35<br>-curvature and torsion implications 2 interactive video<br>-begin assessment guide | curvature and torsion<br>helix and strake | -prepare for in-class curves assessment<br>-complete any open items |
| 2/14–2/16 | in-class curves assessment | -surfaces, geodesics and coverings interactive video<br>-read pp. 247–250 | covering<br>geodesics<br>cone | -read pp. 67–68, 77–82, 209<br>-coordinates and geodesic curvature interactive video |
| 2/21–2/23 | geodesics<br>sphere | -read pp. 70–76, 212<br>-speed of a geodesic interactive video | geodesics<br>round donut | -read "Surfaces"<br>-first fundamental form interactive video<br>-choose surface for Project 2 |
| 2/28–3/2 | geodesics<br>metric form<br>flat and round donuts | -read pp. 83–87<br>-shape operator interactive video | shape operator | -read pp. 88–91, 91–96, 107–108, 111–114, 123–124<br>-II and Gauss's Theorema Egregium interactive video |
| 3/14–3/16 | $\pi$-day<br>Gauss and mean curvature | -read p. 164<br>-surface area interactive video | surface area<br>matching activity | -read pp. 275–277, 289–292<br>-Gauss Bonnet video<br>-re-engage matching |
| 3/21–3/23 | Gauss Bonnet | -Project 2: research, investigate and present a surface | Project 2 presentations | -read pp. 226–235<br>-surfaces not embedded interactive video<br>-begin assessment guide |
| 3/28–3/30 | surfaces not in $\mathbb{R}^3$<br>Klein bottles<br>hyperbolic | - prepare for in-class surfaces assessment<br>-complete any open items | in-class surfaces assessment | -read pp. 397–416<br>-geodesic equations, tensors and spacetime interactive video |
| 4/4–4/6 | spacetime and metric forms | -read "How to Create Your Own Universe in Three Easy Steps"<br>-Minkowski spacetime and Christoffel computations interactive video<br>-choose metric for Project 3 | Christoffel symbols and geodesics<br>metric form research suggestions | -read pp. 416–430<br>-wormhole metric, curvatures and relativity interactive video |
| 4/11–4/13 | curvatures<br>recording final project suggestions | -begin final project | work on project 3 or final project | -Project 3: research, investigate and present a metric form |
| 4/18–4/20 | Project 3 presentations | -general relativity and the field equations interactive video<br>-read "Relativity" | relativity<br>concluding activities | -course survey<br>-course evaluations |
| 4/25–4/27 | work on final project or optional revisions | -complete any open items | share final project idea or title | - final project video |
| 4/29 | turn in video presentation on ASULearn by the beginning of our 2pm assigned time<br>during the assigned time, conduct video project peer review and self-evaluation<br>(optional) revisions on one in-class assessment, one of the first three projects | | | |