Minkowski SpaceTime Model

Minkowski space, Lorentz geometry, special relativity

$$\begin{bmatrix} t & x & y & z \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$c, c^2, -$

$$g_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$ds^2 = g_{ab} dx^a dx^b \quad ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

where t is time and x, y, z are rectangular coordinates in space. Show that free particles follow straight line geodesics.

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vector is null like or light like if $|v| = 0$:

$$\sqrt{\begin{bmatrix} t & x & y & z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}}$$

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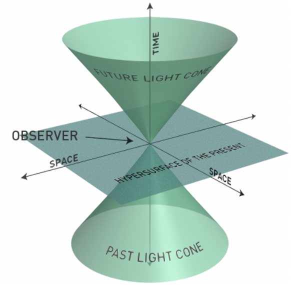
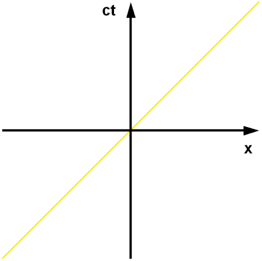
spacetime diagram called light cone: hypercone in $\mathbb{R}^4$ projected

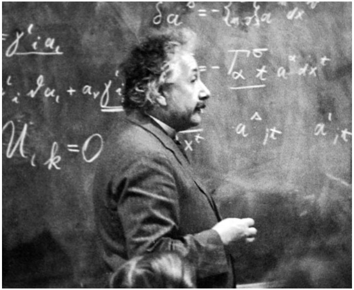




null geodesics: path that a particle without mass travels

time like $v^T g_{ij} v > 0$ space like $v^T g_{ij} v < 0$

How do we find the Christoffel symbols?

Elwin Bruno Christoffel and Albert Einstein

http://www.ethbib.ethz.ch/aktuell/galerie/christoffel/Portr_gross.jpg
http://scienceblogs.com/startswithabang/files/2013/07/einstein.jpg

Rewrite $\vec{x}_{uu}$ by taking uth partial of $E = \vec{x}_u \cdot \vec{x}_u$

$E_u = \vec{x}_{uu} \cdot \vec{x}_u + \vec{x}_u \cdot \vec{x}_{uu} = 2\vec{x}_{uu} \cdot \vec{x}_u$

$\vec{x}_{uu} = \Gamma^u_{uu}\vec{x}_u + \Gamma^v_{uu}\vec{x}_v + lU$, so $\vec{x}_{uu} \cdot \vec{x}_u = \Gamma^u_{uu}\vec{x}_u \cdot \vec{x}_u = \Gamma^u_{uu}E$

Thus $\frac{E_u}{2} = \vec{x}_{uu} \cdot \vec{x}_u = \Gamma^u_{uu}E$ so $\Gamma^u_{uu} = \frac{E_u}{2E}$

Similarly $\Gamma^v_{uu} = -\frac{E_v}{2G}$, $\Gamma^u_{uv} = \frac{E_v}{2E}$, $\Gamma^v_{uv} = \frac{G_u}{2G}$, $\Gamma^u_{vv} = -\frac{G_u}{2E}$, $\Gamma^v_{vv} = \frac{G_v}{2G}$

Minkowski Christoffel symbols?

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What is g_{ij}?
Christoffel symbols

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{db} - \partial_d g_{bc}).$$

geodesics: $\ddot{x}^a + \Gamma^a_{bc} \dot{x}^b \dot{x}^c = 0$

Space-Time Time

- Special relativity with Ralph Alpher, one of the creators of the big bang.

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Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

The Origin of Chemical Elements

R. A. Alpher*
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
AND
H. Bethe
Cornell University, Ithaca, New York
AND
G. Gamow
The George Washington University, Washington, D. C.
February 18, 1948

As pointed out by one of us,[1] various nuclear species must have originated not as the result of an equilibrium corresponding to a certain temperature and density, but rather as a consequence of a continuous building-up process arrested by a rapid expansion and cooling of the

We may remark at first that the building-up process was apparently completed when the temperature of the neutron gas was still rather high, since otherwise the observed abundances would have been strongly affected by the resonances in the region of the slow neutrons. According to Hughes,[2] the neutron capture cross sections of various elements (for neutron energies of about 1 Mev) increase exponentially with atomic number halfway up the periodic system, remaining approximately constant for heavier elements.

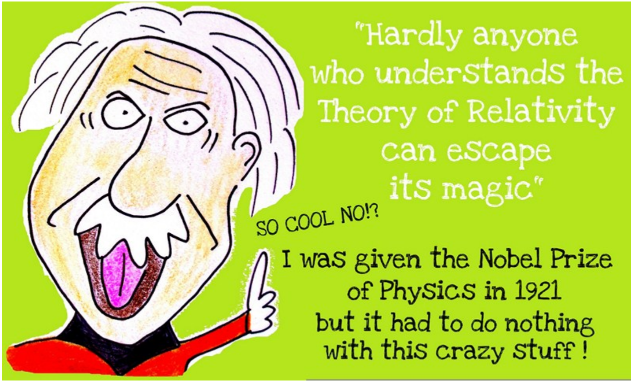
Using these cross sections, one finds by integrating Eqs. (1) as shown in Fig. 1 that the relative abundances of various nuclear species decrease rapidly for the lighter elements and remain approximately constant for the elements heavier than silver. In order to fit the calculated curve with the observed abundances[3] it is necessary to assume the integral of $\rho_n dt$ during the building-up period is equal to 5×10^4 g sec./cm³.

On the other hand, according to the relativistic theory of the expanding universe[4] the density dependence on time is given by $\rho \cong 10^6/t^2$. Since the integral of this expression diverges at $t=0$, it is necessary to assume that the building-up process began at a certain time t_0, satisfying the relation:

$$\int_{t_0}^{\infty} (10^6/t^2)dt \cong 5\times10^4, \qquad (2)$$

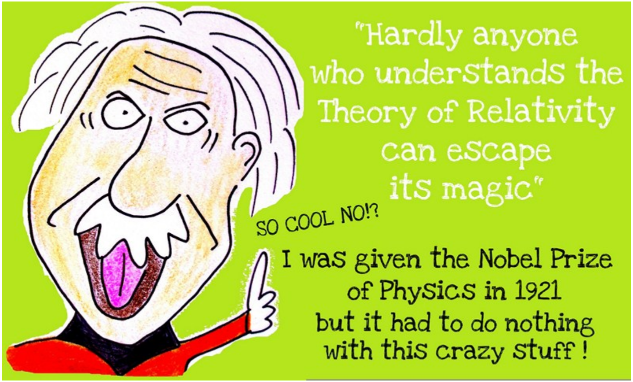
which gives us $t_0 \cong 20$ sec. and $\rho_0 \cong 2.5\times10^5$ g sec./cm³. This result may have two meanings: (a) for the higher densities existing prior to that time the temperature of the neutron

- Riemannian Geometry of Orbifolds PhD



www.thecrazyhistoryofhistory.com/2012/09/the-theory-of-relativity-for-dummies.html

Why should followers of special relativity not be taken seriously?



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Why should followers of special relativity not be taken seriously?

They fail to see the gravity of the situation!

geodesics

$$\ddot{x}^a + \Gamma^a_{bc}\dot{x}^b\dot{x}^c = 0$$

Lagrangian

$$I = g_{ab}\dot{x}^a\dot{x}^b.$$

geodesic will satisfy the Euler-Lagrange equations

$$\frac{d}{ds}\left(\frac{\partial I}{\partial \dot{x}^a}\right) - \frac{\partial I}{\partial x^a} = 0 \text{ for all } a.$$

Once we calculate an equation for each a, we can compare to the geodesic equation in order to read off the Γ^a_{bc} Christoffel symbols, because both the Euler-Lagrange equation and the geodesic equation will be expressed in terms of $\ddot{x}^a$.

$$ds^2 = r^2 d\phi^2 + r^2 sin^2(\phi) d\theta^2$$

a) What is x^1 for $a = 1$?

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b) Write $I = g_{ab}\dot{x}^a\dot{x}^b = r^2\dot{\phi}^2 + r^2\sin^2(\phi)\dot{\theta}^2$

c) Let $a = 1$ in the Euler-Lagrange Equation $\frac{d}{ds}(\frac{\partial I}{\partial \dot{x}^a}) - \frac{\partial I}{\partial x^a} = 0$ for all a. To get used to Einstein summation notation, write out the Euler-Lagrange equation with the $a = 1$ angle:

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g) Repeat to find the four Γ^2_{ab} Christoffel symbols?