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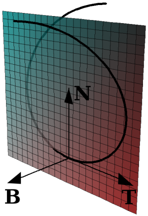

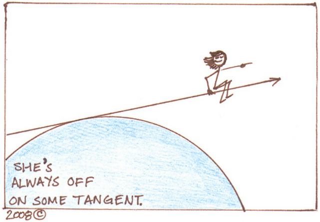

brownsharpie.courtneygibbons.org/wp-content/comics/2008-08-22-off-on-a-tangent.jpg

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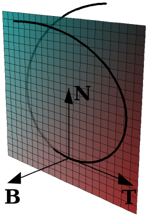

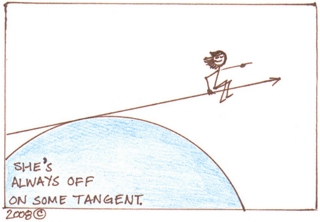

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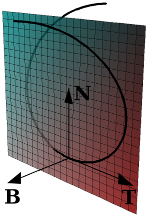

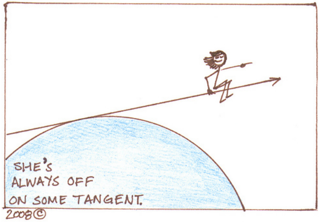

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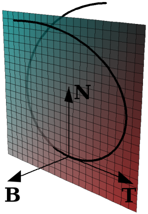
N the Unit Normal (Middle Finger) and $\vec{\kappa}$ Curvature



$T(t) = (-\sin(\omega t), \cos(\omega t), 0)$

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while $\alpha'(s)$ has length 1, $\alpha''(s)$ usually does not so $N = \frac{\vec{\kappa}}{\kappa} = \frac{\vec{\kappa}}{|\vec{\kappa}|}$

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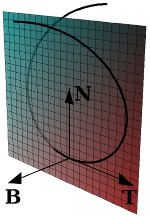

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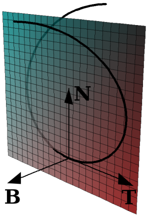

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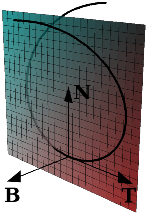

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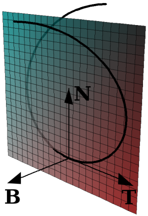

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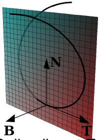
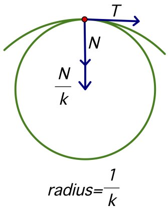
$T \cdot T = 1$, so take the derivative: $T' \cdot T + T \cdot T' = 0$
$2T \cdot T' = 0$. Then $T \cdot T' = 0$ and as long as neither is 0, then they are perpendicular.



Tracking the motion of T via T' tells us how the curve *curves*
T turns towards N. Also, κ tells us how fast T turns
$\vec{\kappa} = \frac{T'(t)}{|\alpha'(t)|}$, $\kappa = |\vec{\kappa}|$, $N = \frac{\vec{\kappa}}{\kappa}$

Osculating Circle



- Best fit circle is the
  osculating circle radius $\frac{1}{\kappa}$ and center $\alpha(t) + \frac{N}{\kappa}$
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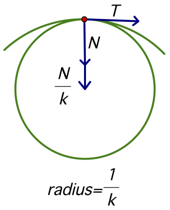
$$T \times N =$$

Osculating Circle



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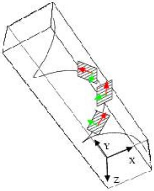
$$N = \frac{\vec{\kappa}}{|\vec{\kappa}|} = (-\cos(\omega t), -\sin(\omega t), 0)$$
$$T \times N = \vec{k} = (0, 0, 1)$$

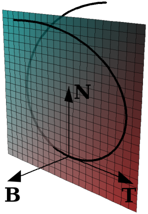
osculating plane $(x - r\cos(\omega t), y - r\sin(\omega t), z - 0) \cdot (0, 0, 1) = 0$

osculating circle center (0,0,0) and radius r

B the Unit Binormal (Thumb)

T and N form a plane, called the osculating plane and B, the binormal, is normal to that plane.





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We unitized other vectors to form T and N. Why is $B = T \times N$ also a unit vector?

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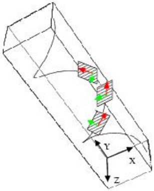

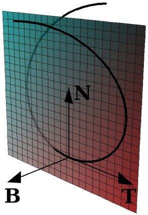

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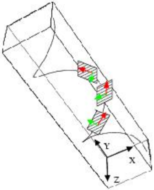

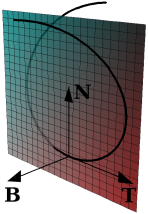

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$|B| = |T| \times |N| = |T||N| \sin\theta = 1 \cdot 1 \cdot \sin 90 = 1$

$B' = -\tau N = (0, 0, 0)$ so $\tau = 0$.