1.3: *TNB: Unit Tangent T (Pointer Finger)*

Example: $\alpha(t) = (r\cos(\omega t), rsin(\omega t), 0)$ with $r, \omega \in \mathbb{R}$ constant

$T = \alpha'(s)$, where $s = \int |\alpha'(t)| dt$ is the arc length

$T = \frac{\alpha'(t)}{|\alpha'(t)|}$. If t is time, then $T = \frac{\vec{v}}{|\vec{v}|} = \frac{\text{velocity}}{\text{speed}}$

Tracking the motion of T tells us how the curve *curves*.

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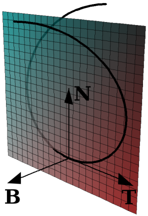
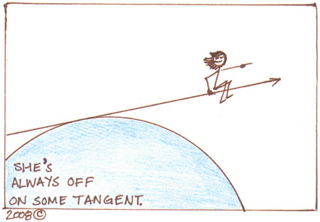
T turns towards N and κ tells us how fast T turns:

$T'(s) = \vec{\kappa} = \kappa N$





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`brownsharpie.courtneygibbons.org/wp-content/comics/2008-08-22-off-on-a-tangent.jpg`

N the Unit Normal (Middle Finger) and $\vec{\kappa}$ Curvature

$\alpha''(s) = T'(s) = \vec{\kappa} = \kappa N$, so $N = \frac{\vec{\kappa}}{\kappa} = \frac{\vec{\kappa}}{|\vec{\kappa}|}$
Note: while $\alpha'(s)$ has length 1, $\alpha''(s)$ usually does not

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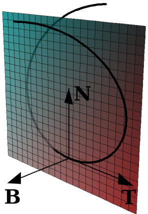
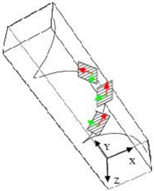
If T is not parameterized by arc length, apply chain rule:
$\vec{\kappa} = \frac{dT}{ds} = \frac{dT}{dt}\frac{dt}{ds} = \frac{\frac{dT}{dt}}{\frac{ds}{dt}} = \frac{T'(t)}{|\alpha'(t)|}$
Why is T' perpendicular to T? (this shows N is too)

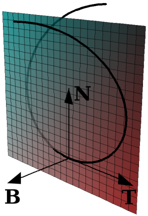


http://www.sciencecartoonsplus.com/gallery/physics/

B the Unit Binormal (Thumb)

T and N form a plane, called the osculating plane and B, the binormal, is normal to that plane.





http://cs-www.cs.yale.edu/homes/li-gang/research/CurveStereo/index.html, CC-BY-SA-3.0 Salix alba at English Wikipedia

We unitized other vectors to form T and N. Why is $B = T \times N$ also a unit vector?

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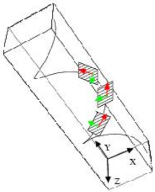
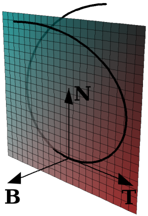




We unitized other vectors to form T and N. Why is $B = T \times N$ also a unit vector?

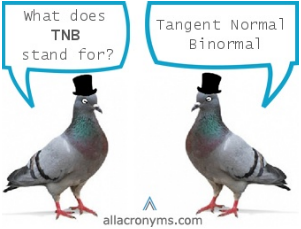
$B' = -\tau N$

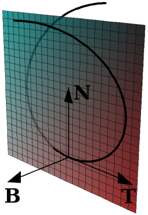
As your hand moves along a curve, rotate it so the thumb (B) turns away from the middle finger N ($-N$) with a speed of τ. B' captures the movement of the osculating plane.

Frenet-Serret Frame *TNB*

- $T = \alpha'(s) = \frac{\alpha'(t)}{|\alpha'(t)|}$. If t is time, then $T = \frac{\vec{v}}{|\vec{v}|} = \frac{\text{velocity}}{\text{speed}}$
- $N = \frac{\vec{\kappa}}{|\vec{\kappa}|} = \frac{\vec{\kappa}}{\kappa}$

  where $\vec{\kappa} = \alpha''(s) = T'(s) = \frac{dT}{ds} = \frac{dT}{dt}\frac{dt}{ds} = \frac{\frac{dT}{dt}}{\frac{ds}{dt}} = \frac{T'(t)}{|\alpha'(t)|}$
- $B = T \times N$

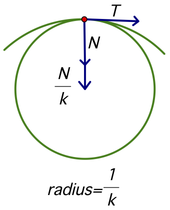




http://www.allacronyms.com/TNB/Tangent-Normal-Binormal, CC-BY-SA-3.0 Salix alba at English Wikipedia

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

Osculating Circle



- Best fit circle
- https://faculty.evansville.edu/ck6/GalleryTwo/CK_Frenet_Osculating_A.gif
- Historical curves
  http://mathshistory.st-andrews.ac.uk/Curves/Curves.html
  http://mathworld.wolfram.com/Astroid.html
- Geometric intuition:
  http://theronhitchman.blogspot.com/2015/02/the-geometry-of-frenet-serret-equations.html

1. Which of the following is $\alpha'(s)$ where s is the arc length parameter?

a) velocity

b) unit tangent vector T

c) curvature vector

d) more than one answer works

e) none of the above

2. Which of the following is $\alpha''(s)$ where s is the arc length parameter?

a) acceleration
b) jerk
c) curvature vector
d) unit normal vector N
e) more than one answer works