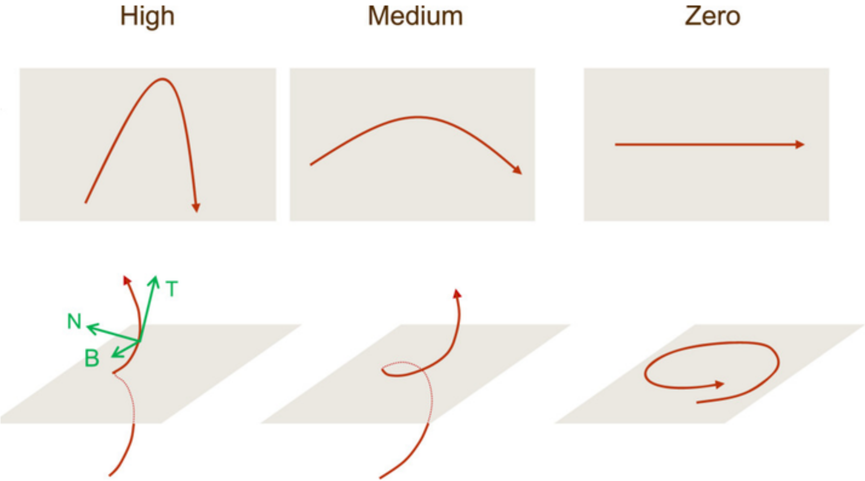
- Prove that $\alpha(s)$ is a line $\Leftrightarrow \kappa = 0$
- Prove that $\alpha(s)$ is planar $\Leftrightarrow \tau = 0$



Pitt et al.: "Polyphony: superposition independent methods for ensemble-based drug discovery." *BMC Bioinformatics* 2014 15:324.

http://cs.appstate.edu/~sjg/class/4140/tactivitiescurves2.pdf

Helix

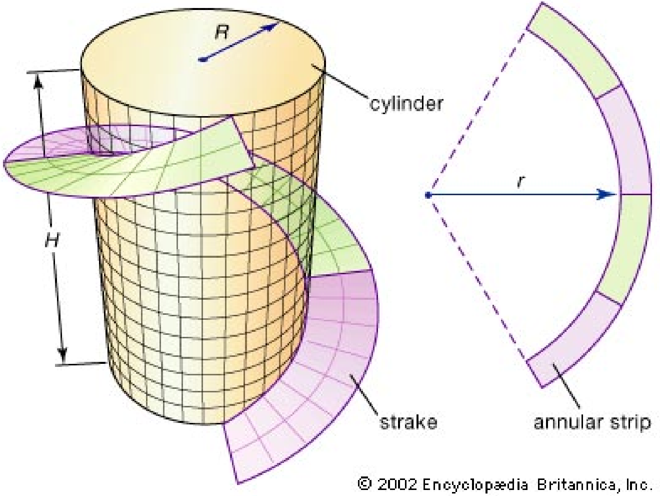
http://www.nerdytshirt.com/calculus3-tshirts.html

Helix

http://www.nerdytshirt.com/calculus3-tshirts.html

Notice that $\dfrac{\tau}{\kappa}$ is constant.

Strake

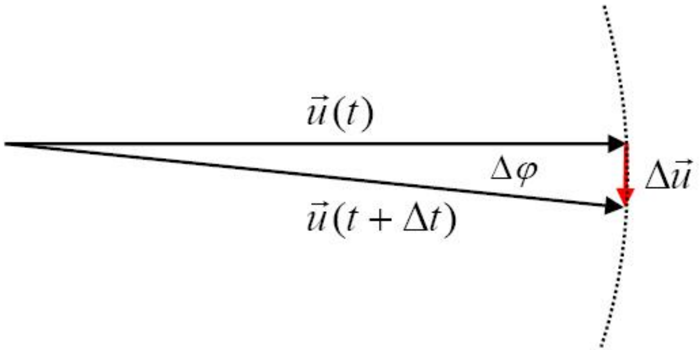


https://cdn.britannica.com/22/70822-004-B85BF4BD/

strake-strip-dimensions-cylinder-contour-Techniques-differential.jpg

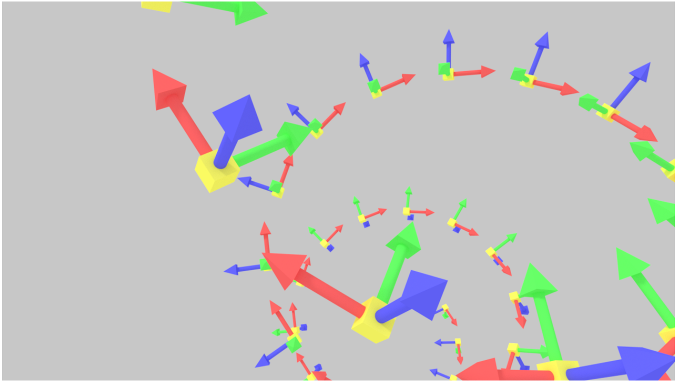
1. To prove that the derivative of a unit vector $\vec{u}$ is perpendicular to the original...

a) take the derivative of $\vec{u} \cdot \vec{u}$ and argue from there
b) take the derivative of $\vec{u} \times \vec{u}$ and argue from there
c) both of the above
d) none of the above



2. Which of the following represents $-\kappa T + \tau B$?

a) T'
b) N'
c) B'
d) more than one of the above
e) none of the above



https://janakiev.com/blog/framing-parametric-curves/

Blog post on creating tubes, ribbons and moving camera orientations

3. Why is N perpendicular to T?

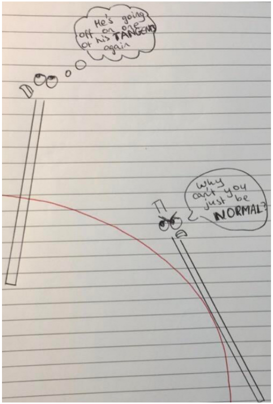
a) Because N is parallel to $\vec{k}$, and $\vec{k}$ is the derivative of the unit vector T and hence perpendicular to it

b) Because $N = B \times T$

c) both of the above

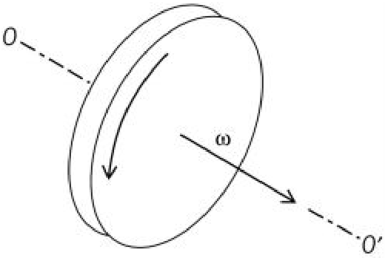
d) It isn't perpendicular

e) It is perpendicular but not by any of the above



https://www.reddit.com/r/mathmemes/comments/e8t8u3/maths_pun_comedy/

4. In the following image, if a coaster car is traveling for a bit on a coaster shaped like the following, following the curve, then

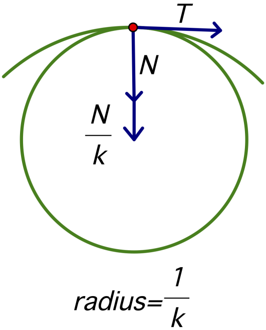


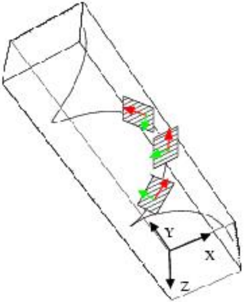
http://img.tfd.com/mgh/cep/thumb/Angular-velocity-shown-as-an-axial-vector.jpg

a) the people in the coaster would feel the curvature of the curve as a tilt, dip or even flip upside down
b) the people in the coaster would feel the curvature pulling them sideways
c) both of the above
d) none of the above

Osculating Plane and Osculating Circle

curvature k: tracking T & how the curve *curves*
$-$torsion τ: tracking B & twists out of osculating plane





http://cs-www.cs.yale.edu/homes/li-gang/research/CurveStereo/index.html

osculating circle: radius $\frac{1}{k}$ and center $\alpha(t) \pm \frac{1}{k}N$
osculating plane: $((x, y, z) - \alpha(t)) \cdot B(t) = 0$

Frenet-Serret Frame *TNB*

- $T = \alpha'(s) = \frac{\alpha'(t)}{|\alpha'(t)|}$. If t is time, then $T = \frac{\vec{v}}{|\vec{v}|} = \frac{\text{velocity}}{\text{speed}}$
- $N = \frac{\vec{\kappa}}{|\vec{\kappa}|} = \frac{\vec{\kappa}}{\kappa}$
  where $\vec{\kappa} = \alpha''(s) = T'(s) = \frac{dT}{ds} = \frac{dT}{dt}\frac{dt}{ds} = \frac{\frac{dT}{dt}}{\frac{ds}{dt}} = \frac{T'(t)}{|\alpha'(t)|}$
- $B = T \times N$
  $B'(s) = \frac{B'(t)}{|\alpha'(t)|} = -\tau N$
  As your hand moves along a curve, rotate it so the thumb (B) turns away from the middle finger N ($-N$) with a speed of τ. B' captures the movement of the osculating plane $((x, y, z) - \alpha(t)) \cdot B(t) = 0$.

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$