

# In-class Timed Assessment: Curves

It is time for our first in-class assessment, on curves, in order to be sure that everyone reviews some of the fundamental concepts before we move on to surfaces.

## During Class

- You may make yourself some reference notes on both sides of the very small card I hand out. The mini-reference card must be handwritten. Think of the card as a way to include some important concepts, computations, or derivations that you aren't as comfortable with. You won't have room for much, so you should try to internalize as much as you can.
- You may have standalone ear plugs—no technological connections connected to the internet though.
- This assessment has an individual component as well as a component where you can work in groups.

You work alone until I collect the individual portion and say it is “group time” and time to turn in the individual portion. Then you may continue to work alone or in groups (or a combination!). The idea is to give you opportunities to communicate course content with your peers, since this is one of ASU's main educational goals: “Successful communicators interact effectively with people of both similar and different experiences and values.” The only guidelines are that each person must eventually write up and turn in their own, the only resources you are allowed to use is each other, and you should spend the time inside the classroom effectively engaging.

If you finish the individual component early, proceed with the group component on your own until I announce group time—the idea is to have silence for a good portion of class before we switch to “group time.” If you finish the entire assessment early, then you may leave early.

- Your grade will be based on the quality and depth of your responses in the timed environments. Partial credit will be given, so (if you have time) showing your reasoning or thoughts on questions you are unsure of can help your grade.

## Review Suggestions

Be sure you could respond to questions on these. I want you to understand the material and I am happy to help!

**Short Derivations/Proofs** Be able to prove the following in the language of our class:

- Prove the derivative of a unit vector  $\vec{u}$  is perpendicular to the original vector if neither are  $\vec{0}$ .
- Prove  $B$  is a unit vector. You may assume that  $T$  and  $N$  are unit length and perpendicular to each other, and that  $B$  is defined in terms of them.
- The proofs of the TNB derivative equations. You would be given one short part and some underlying assumptions:
  - Prove  $B'$  has no component in the  $T$  direction. You may assume that the Frenet equation about  $T'$  holds but not the other two. You may also assume cross product relationships and dot product relationships between  $T, N$  and  $B$ .

- Prove that  $N'$  has a  $-\kappa$  component in the  $T$  direction. You may assume that the Frenet equations about  $T'$  and  $B'$  hold but not the one about  $N'$ . You may also assume cross product relationships and dot product relationships between  $T, N$  and  $B$ .
- Prove that  $N'$  has a  $\tau$  component in the  $B$  direction. You may assume that the Frenet equations about  $T'$  and  $B'$  hold but not the one about  $N'$ . You may also assume cross product relationships and dot product relationships between  $T, N$  and  $B$ .
- Prove that  $\alpha(s)$  with  $\kappa = 0$  is a line.

You should know the results of other statements too, but I won't ask you for any other complete derivations, other than the ones above.

### Calculations and Interpretations

By-hand computations and interpretations, like

- Finding  $T, \vec{\kappa}$ , and  $\kappa$  curvature for a curve
- Finding  $N$ , given  $T$
- Finding  $B$  and  $\tau$ , given  $T$  and  $N$
- Interpreting results and applications of the Frenet frame, like recognizing that a line is the shortest distance curve between two points in Euclidean geometry, a  $\tau = 0$  curve is planar, a  $\kappa = 0$  curve is a line, a curve with constant positive scalar curvature that is planar is part of a circle, a curve with constant  $\frac{\tau}{\kappa}$  is a cylindrical helix...

### Fill in the Blank/Short Answer

There will be some short answer questions, such as:

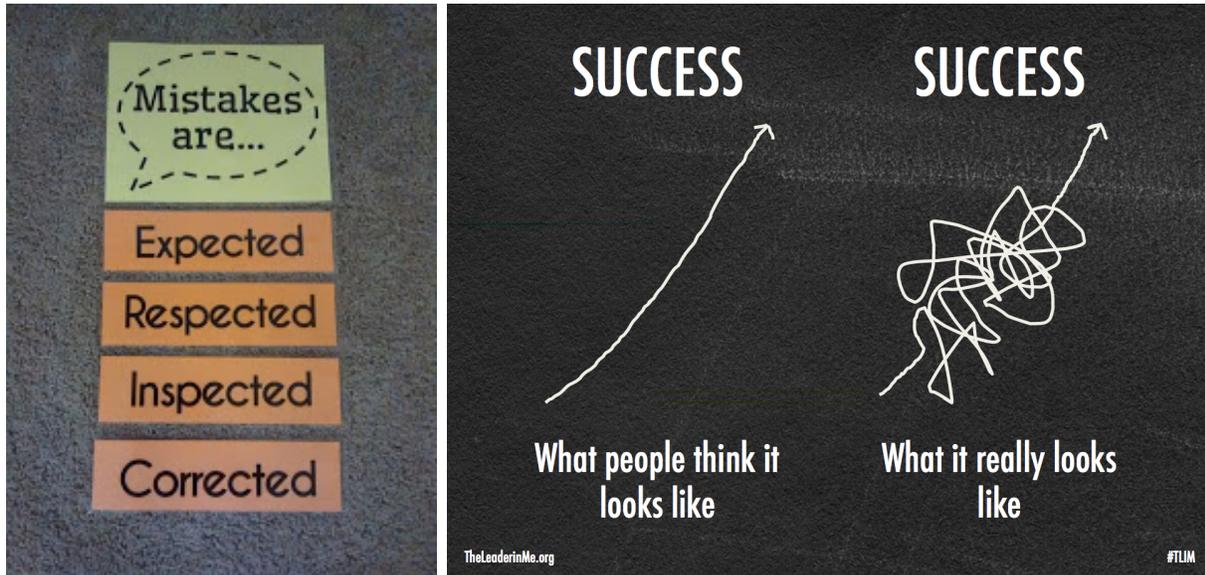
- questions similar to previous polling questions, video interactions, matching activity questions, or other activities from class where you fill in a blank instead. For instance,
  - $-\kappa T + \tau B =$  \_\_\_\_\_
  - $N' =$  \_\_\_\_\_
  - unit vector that lies along the direction which the curve is currently bending in = \_\_\_\_\_

As you can see here, there is often more than one answer possible for fill in the blank questions: choose one response. Full credit responses demonstrate deep understanding of differential geometry. For instance, here you could fill in  $-\kappa T + \tau B = N'$  for the first response,  $N' = -\kappa T + \tau B$  for the second, and  $\frac{T'(t)}{|\alpha'(t)|}$  for the third, among other possible responses. On the other hand, responding with  $N' = N'$ , while a true statement, doesn't demonstrate deep understanding of differential geometry. Informal responses are fine as long as they are correct and demonstrate understanding of the material from activities inside and outside of class.

- examples, such as a curve with  $\kappa = 0$ , a curve with nonzero constant  $\kappa$ , a curve with  $\tau = 0$ , a curve with nonzero constant  $\tau$ ...
- parametrizations, curvature or torsion of "basic" curves such as a line, circle, or helix

## (optional) Revision Opportunity for the Curves or Surfaces Assessment

In many real-life scenarios, such as exams in some graduate schools (e.g., in my experiences at the University of Pennsylvania), in actuarial science exams, edTPA for teacher certification, and much more, an opportunity to retake an assessment or revise part of it is allowed or even quite common. At Penn we could retake our preliminary exam after a semester had passed but the oral exam was a one-shot deal.



<https://mathequalslove.blogspot.com/p/free-classroom-posters.html>

<https://www.leaderinme.org/blog/the-power-of-a-growth-mindset/>

To encourage in-class assessments as a learning experience and to accommodate for emergencies, like missing the assessment completely, you can revise **one** of them to replace the original. There are no make-up assessments but there is flexibility in this manner.

The idea is to turn any mistakes into productive ones (or possibly even enlightening moments where you made some additional connections and hopefully have some aha moments!). Making mistakes is integral to the learning process as long as you review and understand any misconceptions, and I want to encourage and reward this. For the revision, I expect you to use online resources and get help from me. You can annotate your original graded assessment, writing your corrections on it and/or write on a separate sheet of paper. Regardless, be sure to turn back in the original too. I want you to solidify the material and I am here to help!