Differential Geometry Project 1: Curves

You may work alone or in a group of up to 3 people and turn in one per group.
The Applets can be found on the main web page.

(1 of 3) Choose one of the following three problems.

1. **Problem 1**: Witch of Agnesi $\alpha(t) = (2t, \frac{2}{1+t^2}, 0)$

Enter the curve into the Maple Applet TNBapplet.mws:
(2*t, 2/(1+t^2), 0) t ranges from -1 to 1

Part A: Sketch the curve and the Frenet Frame at two different places on the curve.
Part B: Search for the Witch of Agnesi and summarize what you found in your own words.

2. **(or) Problem 2:** Lemniscate of Bernoulli $\alpha(t) = (\frac{3\cos(t)}{1+\sin^2 t}, \frac{3\sin(t)\cos(t)}{1+\sin^2 t}, 0)$

Enter the curve into the Maple Applet TNBapplet.mws:
(3*cos(t)/(1+sin(t)*sin(t)) , 3*sin(t)*cos(t)/(1+sin(t)*sin(t)) , 0) t ranges from -2*Pi to 2*Pi

Part A: Sketch the curve and the Frenet Frame at two different places on the curve.
Part B: Search for the Lemniscate of Bernoulli and summarize what you found in your own words.

3. **(or) Problem 3:** Vivani's Curve $\alpha(t) = (1+\cos(t), \sin(t), 2\sin(\frac{t}{2}))$

Enter the curve into the Maple Applet TNBapplet.mws:
(1+cos(t), sin(t), 2*sin(t/2)) t ranges from -2*Pi to 2*Pi

Part A: Sketch the curve and the Frenet Frame at two different places on the curve.
Part B: Search for Vivani's Curve and summarize what you found in your own words.

(4 of 4) Complete problems 4-7

4. **Problem 4:** Cycloid $\alpha(t) = (t+\sin(t), 3-\cos(t), 0)$

Enter the curve into the Maple Applet TNBapplet.mws:
(t+sin(t), 3-cos(t),0) t ranges from 0 to 7

Part A: Sketch the curve and the Frenet Frame at two different places on the curve.
Part B: Is the Frenet Frame defined everywhere? If not, specify any problem points, and explain whether any of the vector components of the Frenet Frame are defined at the problem points. For those vector components that are not defined, explain why not. Check your assertions by testing out various t values.
Part C: Why is the cycloid interesting from a physics standpoint? Use a web search, but write this up in your own words.

5. **Problem 5:** Spiral $\alpha(t) = (3\cos(t), 3\sin(t), \log(t))$

Enter the curve into the Maple Applet TNBapplet.mws:
(3*cos(t), 3*sin(t),log(t)) t ranges from .0000001 to 2*Pi

Part A: Sketch the curve and the Frenet Frame at two different places on the curve.
Part B: Is the Frenet Frame defined everywhere? If not, specify any problem points, and explain whether any of the vector components of the Frenet Frame are defined at the problem points. For those vector components that are not defined, explain why not. Check your assertions by testing out various t values.

6. **Problem 6:** Choose your favorite planar curve. Write down the parametric version of the curve. Use the Maplet spacecurve.mw to find the Plot, Velocity, Acceleration, Jerk, Speed, ArcLength, Curvature, and Torsion, and write down what Maple finds. Next explain in your own words what each of the pieces means physically and/or geometrically, and specifically relate your answers to this example. You may use a web search and/or the text to help you.

7. **Problem 7:** Choose your favorite non-planar curve. Explain how I know that the torsion will be non-zero. Write down the parametric version of the curve. Use the Maplet spacecurve.mw to find the Plot, Velocity, Acceleration, Jerk, Speed, ArcLength, Curvature, and Torsion, and write down what Maple finds and explain by specifically relating your answers to this example.

****Challenge Problem** (Extra Credit)

8. Computer Images of Curves

In Maple, use plot commands to plot the planar curves

y = sqrt(10^(-30) +x^2) and y = |x|

When plotted from x=-1..1, the curves look like they behave the same at the origin. Test out smaller ranges of x (x=-1/10..1/10, etc).

Can you distinguish the behavior of these curves at the origin by a similar Maple plot command? If so, what is the largest value of x, of the form x=-1/100...0, … 1/100...0 necessary to accomplish this?

Can you distinguish the behavior of the curves at the origin using differential geometry techniques? If so, explain how, and show work to distinguish the curves at the origin.