Project 2: Research, Investigate, and Present a Surface

You may work alone or with one other person and turn in one per group. Surfaces are on a **first come-first-served** basis in the ASULearn choice selection feature. If you are working in a group, one person selects the surface and the other selects "working with someone else who already selected our topic."

Explore the following questions via the Maple file I provided for you as well as researching and analyzing yourself. (**Keep track of ALL your references for # 25**). **Write it up in your own words** but you may use pictures from elsewhere (with proper reference).

You will turn in all of the following and share with your classmates (see #27).

1. List your preferred first name(s). If you are turning this in with a partner, list both names.

2. Search an image database for "differential geometry of *", where * is a name of your surface that you selected in the choice feature on ASULearn. You might use Google images, for instance. Provide one or more interesting images that relate. Be sure to list any picture references (and any other references) in #25. Google is a database, not typically the original source of an image, so be sure to track back to the original source.

3. Handwrite or professionally typeset general formulas for the following entities as a review in equations and/or words. Assume that you have a surface parameterized as $X(u, v)$. Do NOT do any calculations for your specific surface here, but do show generic formulas connected to the language of our class and/or explain how to calculate each from a parameterization of the surface or curve itself. Your answers may build upon one another, i.e. using part (a) in another part.

   (a) Unit Normal U to a surface $X(u, v)$
   (b) Curvature of a curve $\gamma(t)$ on $X(u, v)$
   (c) Normal Curvature of $\gamma(t)$ on $X(u, v)$
   (d) Geodesic Curvature of $\gamma(t)$ on $X(u, v)$
   (e) a curve is a geodesic if the geodesic curvature is...
   (f) E (of a surface $X(u, v)$)
   (g) F
   (h) G
   (i) l
   (j) m
   (k) n
   (l) Gauss Curvature K of a surface $X(u, v)$
   (m) Mean Curvature of H a surface $X(u, v)$
   (n) Shape operator $S(X_u)$
   (o) Shape operator $S(X_v)$

4. Adapt the Maple file diffgeomproj2.mw and follow the instructions there. At the bottom of the file, I have listed parameterizations for your surface. Move the g command for your surface to the top of the file, just under the packages, and delete the others. Follow the instructions and execute the rest of the commands to obtain output for your surface. You will collate the Maple output for your surface into the PDF you submit, so be sure to save any modifications.

5. In the section on geodesic and normal curvatures, find a curve on your surface that is NOT a geodesic. You may need to also modify other code, like to remove items in the second display command to see the curvature vectors on their own, or change the limits on the coordinates for the domain of the surface or curve. You'll want to rotate the 2 display command outputs so that they are revealing before you create your PDF.

6. Next, explore and modify the yes a geodesic section to provide new values and visualizations for a curve that is a geodesic, or as close to a geodesic as you can get. Some good things to start with are
   f1 as 1 and f2 as t
   f2 as t and f1 as 1
   f1 and f2 both as t
   Make uses of the numerical approximation of geodesic curvature below the graphs too. If you can't find a geodesic, how close can you get?

7. Can you say anything about what does a u coordinate curve look like on your surface? For instance, is it recognizable as circles, helices, lines, or other curves we've previously explored? Explain/show work and try to provide the name of the coordinate curves (e.g., tractrix...). If you can't tell from exploring the surface parameterization at first, try selecting a revealing or especially simple value of a constant v to help you explore that specific u coordinate curve. You can also use the Maple file to help you visualize, but be sure to connect the visualization to the parameterization of the curves. Explain your reasoning.

8. Can you say anything about what does a v coordinate curve look like on your surface? Explain your reasoning.

9. Next, use the Maple file to calculate E, F and G for your surface in the First Fundamental Form section.

10. Is $F = 0$ typical for most points on your surface? In addition, whether yes or no, what does that tell us? Explain.

11. Handwrite or professionally typeset the metric form ds^2 for your surface (or the start of it if it is too long/unwieldy!)

12. Compare and contrast the metric form to the flat Euclidean metric form $ds^2 = du^2 + dv^2$. Does the Pythagorean theorem hold for your surface? If not, is the surface hypotenuse too long, too short, or a mixture? Explain your reasoning.

13. Set up, but do not solve, a surface area integral using our formula involving E, F and G. Explain what your limits of integration would be to find the surface area for the entire surface. In addition, if your surface has infinite surface area, cap it off somewhere, and explain what the limits would be and what the capped picture would look like. [For instance for a parameterization of the xy plane $[u, v, 0]$, u and v range from $-\infty$ to ∞, but a capped version of the plane could be a square with u and v from -1..1]

14. Next, use the Maple file to calculate and explore Gauss curvature, Mean curvature, II, the second fundamental form, and the shape operator.

15. What kind(s) of Gauss curvature is possible on your surface (positive, negative, zero) and where on the surface are these possible? Answer and explain/show reasoning, like the numerical approximations of Gauss curvature in the Maple file.

16. Choose at least one revealing point and consider the surface normal at the point and the principal curvatures, the maximum and minimum normal curvatures. For each revealing point, sketch 2 by-hand pictures—one for each principal curvature—that includes part of the surface, the curve, surface normal, osculating circle, and radius. Samples could be like the following, but sketch them for your surface to showcase its geometry.

http://brickisland.net/cs177/?p=144
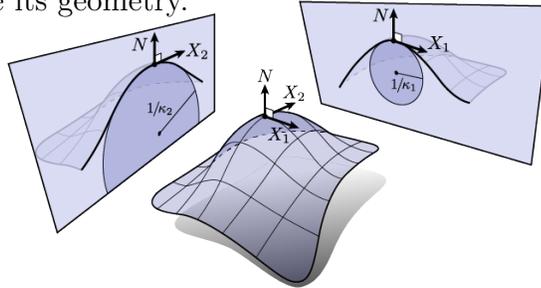
17. Discuss Gauss curvature **intuition** for your surface. Argue geometrically to explain your sketches as well as the positive or negative sign or zeroness of the Gauss curvature.

18. Is the mean curvature for your surface typically 0, i.e. a minimal surface? If not, is the mean curvature typically positive or negative, or a mixture? Answer and explain/show reasoning, like the numerical approximations of mean curvature in the Maple file.

19. Explore the shape operator acting on X_u in the Maple file. Is the rate of change of the surface normal U in the u direction a multiple of either $\vec{x}_u$ or $\vec{x}_v$ individually? If so, which. Or does this application of the shape operator require a linear combination of both tangent vectors?

20. Explore the shape operator acting on X_v in the Maple file. Is the rate of change of the surface normal U in the v direction a multiple of either $\vec{x}_u$ or $\vec{x}_v$ individually? If so, which. Or does this application of the shape operator require a linear combination of both tangent vectors?

21. Search MathSciNet

https://library.appstate.edu/find-resources/databases/subject/mathematical-sciences

for a journal article related to your surface. Modifying the search terms in MathSciNet can make a huge difference. This include minor modifications in plural or not, or possessives or not, as well as Anywhere versus Title searches. Choose one you find interesting and write down the full bibliographic reference from the MathSciNet database. If you can't find anything on MathSciNet, it may be that your surface is more relevant in applications, so you can try another database or Google Scholar.

What is MathSciNet? Historically, mathematicians communicated by letters, during visits, or by reading each other's published articles or books once such means became available. For example, Marin Mersenne had approximately 200 correspondents. Some mathematical concepts were developed in parallel by mathematicians working in different areas of the world who were not aware of each other's progress. In an effort to increase the accessibility of mathematics research articles, reviews began appearing in print journals like Zentralblatt fur Mathematik, which originated in 1931, and Mathematical Reviews, which originated in 1940. Since the 1980s, electronic versions of these reviews have allowed researchers to search for publications. MathSciNet,

the electronic version of Mathematical Reviews, is "the authoritative gateway to the literature of Mathematics" and currently contains over 4 million items.

22. In bullet point format, summarize the *significance* of your surface including (if possible) real-life applications or connections as well as the earliest year you can find related to your surface. Keep track of your sources. To find real-life applications, searches in Google scholar like "engineering Catalan surface" can result in very interesting connections, although one has to be careful with this surface in particular since some sources refer to Catalonia rather than the intended surface. Similarly there are many interesting connections in computer science and more to many of your surfaces.

23. Research mathematicians, physicists, engineers, or others who are related to your surface. These can be people who laid groundwork on the surface or conducted peripheral but connected work. Then search for additional information on one person (or more than one if you prefer), like in MacTutor. If your surface is named for someone, research that person. Regardless, try to select a person with a substantial contribution and connection to your surface. Report back here and keep track of your sources for #25. For instance, perhaps you can find and summarize

    - their contributions or connections to your surface
    - the title of their publication that included content related to your surface or a year or a range of years they worked on your surface, if possible. If of not, then their year of birth and, if applicable, death, would provide their working years
    - what country they worked in
    - something you found interesting about the person

24. In bullet point format, summarize the physically interesting features of your surface.

25. Give proper credit to any references you used, including proper credit to image sources. If you used Wikipedia for anything other than pictures, try to find the original scholarly sources, as Wikipedia notes: "because it can be edited by anyone at any time, any information it contains at a particular time could be vandalism, a work in progress, or just plain wrong"' (accessed 1 January 2022
`https://en.wikipedia.org/wiki/Wikipedia:Wikipedia_is_not_a_reliable_source`).

26. Collate your work into one PDF for submission to the ASULearn assignment for Project 2. **If you are a graduate student**, collate the graduate problems too. Electronically, you can append PDFs you create from Maple to the end of your other PDFs, like by using Preview on a Mac or PDFsam on a PC. Maple lists instructions for creating a PDF. If you have a phone or tablet, apps like Adobe Scan or CamScanner can work well to scan work to one full size multipage PDF. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help.

27. **Elevator pitch presentation about your surface**—up to 2 minutes. The idea of an elevator pitch is to make a short, persuasive pitch that sparks interest in a topic during the time it takes to ride an elevator with a stranger. You will present your elevator pitch during class and each person is limited to 2 minutes. Start your pitch with your name.

    If you work in a group then you should try and pitch different information than your partner did. You could pitch separately or take turns within the same pitch, but both people must speak about equally—as a group you are limited to 4 minutes total.