Homework 1: Review for Curves in Differential Geometry

You may work alone or in a group of at most 2 people, and turn in one per group. You should be prepared to present any of the problems your group turns in. Show reasoning and work—each group must write up their work in their own words. You do not need to simplify your answers, but you do need to show by-hand computations. Try all the problems, but carefully write up to turn in this number:

4140 students: turn in 8 of the 13 problems. Grad students: turn in 12 of them.

The purpose of homework is to learn and practice computational strategies, concepts, and develop critical thinking and problem-solving skills, so you should try problems on your own. Feel free to talk to me or each other if you are stuck on this assignment, but be sure to acknowledge any sources other than your group members or me, including other classmates like "The insight for this solution came from a conversation with Joel." If you use any external sources outside of your group then cite them. If you know how to do a problem and are asked for help outside of your group, try to give hints rather than the solution.

1. Write an equation for the line connecting the points $(-3, 2, 5)$ and $(1, -2, 4)$.
2. Find an equation of the tangent plane to the surface $z = y^2 - x^2$ at the point $(-4, 5, 9)$.
3. Let $f(x, y) = 1 + 2x\sqrt{y}$. Find the directional derivative at the point $(3, 4)$ in the direction of $\vec{v} = \langle 4, -3\rangle$.
4. Wile E. Coyote leaps off a ledge as the Road Runner zips past below him on a perfectly level road. Wile's Acme jetpack propels him in the direction of the positive y axis with an acceleration of 20 m/s^2, and gravity in Wile's cartoon world is 5 m/s^2 in the direction of the negative z-axis, so his acceleration is given by the vector $\langle 0, 20, -5\rangle$. Wile's jump was from 10 meters above the origin with an initial velocity of 2m/s along the positive $\vec{x}$ axis, so his initial velocity is given by the vector $\langle 2, 0, 0\rangle$ and his initial position by the vector $\langle 0, 0, 10\rangle$. What is Wile's position after two seconds?
5. Find a tangent vector to the curve $\langle 2\cos(t), 3\sin(t), t\rangle$ at $t = 2$.
6. Find the speed of the curve $\langle \cos(t), \sin(t), 5\rangle$ at $t = 2\pi$ and the arc length from 0 to 2π. Is $(1, 1, 5)$ on the curve? Why or why not?
7. Find a unit vector pointing in the direction of $\langle 4, 3, -1\rangle$.
8. Given $\vec{v} = \langle 2, 1, 6\rangle$ and $\vec{w} = \langle 3, 1, -1\rangle$, calculate
   (a) $\vec{v} - \vec{w}$
   (b) $\vec{v} + \vec{w}$
   (c) $\vec{v} \cdot \vec{w}$
   (d) $\vec{v} \times \vec{w}$
   (e) $\arccos(\frac{\vec{v}\cdot\vec{w}}{|v||w|})$
   (f) $(\frac{\vec{v}\cdot\vec{w}}{|w||w|})\vec{w}$
9. What are the geometric or physical interpretations of each of the calculations in the last problem? Of two vectors whose dot products are zero?
10. Find a vector that is perpendicular to both $\vec{v} = \langle 2, 1, 6\rangle$ and $\vec{w} = \langle 3, 1, -1\rangle$.
11. The curvature of $\langle t, t^2\rangle$ at t, i.e. of the plane curve $y = x^2$.
12. The planes $x - 2y + 3z = 5$ and $2x + z = 3$ intersect in a line. Parameterize this line.
13. Can every vector in $\mathbb{R}^3$ be written as a linear combination of $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}7\\8\\9\end{bmatrix}$? Why or why not?