Should the Frenet frame be named only for Frenet?

a) yes
b) no, it should include him, but not only Frenet
c) no, strike his name and use a different one

http://1.bp.blogspot.com/-LgzYokAoe_I/VJW9enlNqkI/AAAAAAAAUjY/YYrGH7TPBBY/s1600/baby-name-surprised.jpg

Curves → Surfaces

- The embeddings make a difference as we'll see when we examine curves on other kinds of surfaces (e.g., helix on cylinder versus cone) and in spacetime.
- Given a fixed piece of string, what figure bounds the largest area?

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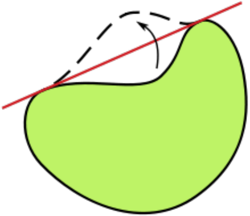

https://upload.wikimedia.org/wikipedia/commons/thumb/0/03/Isoperimetric_inequality_illustr1.svg/440px-Isoperimetric_inequality_illustr1.svg.png

Green's Theorem $\int_\alpha Ldx + Mdy = \int\int \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} dA$

Think Green!

1.6: Isoperimetric Inequality (Schmidt, 1938)

$\alpha(s) = (x(s), y(s))$ closed curve with length L
enclosing region $A \Rightarrow L^2 \geq 4\pi A$
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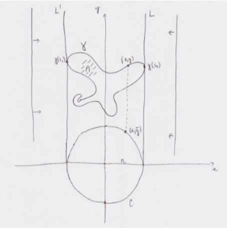
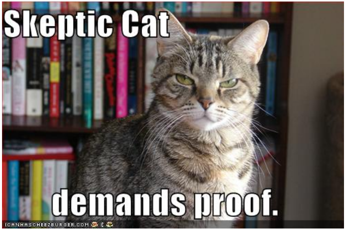
once around? $0 \leq s \leq L$

- Bound by 2 parallels lines
- Circle is stepping stone to go from area to length
- r drops out at the end so specific projection doesn't matter





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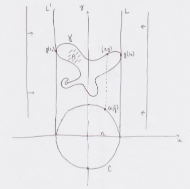
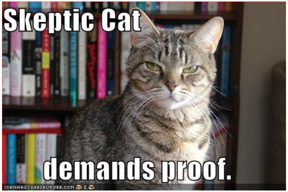

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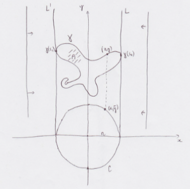
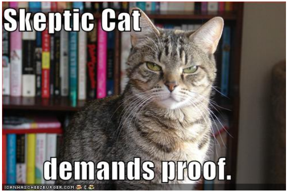

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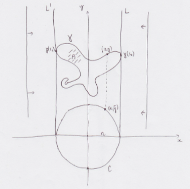
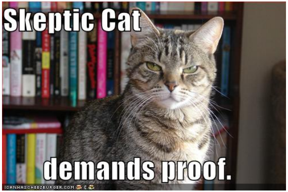

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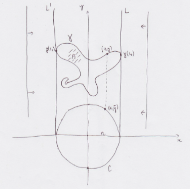
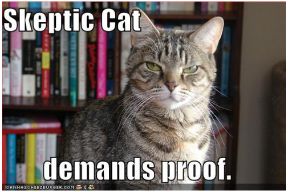

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We'll add these & reduce to get a nice formula for total area: rL

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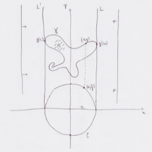
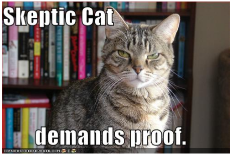

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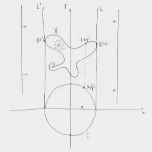

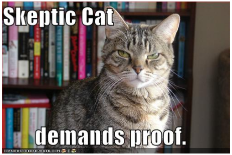

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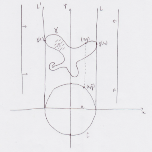

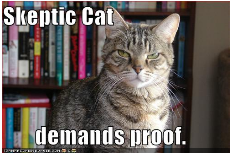

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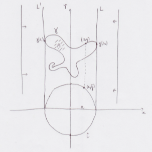
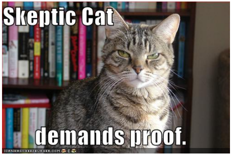

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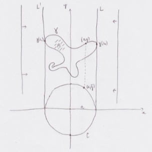

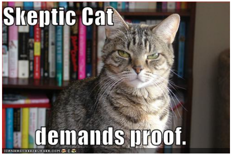

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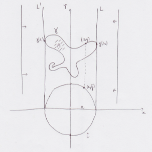
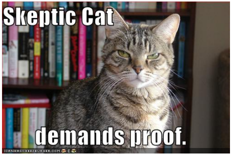

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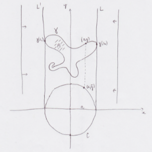

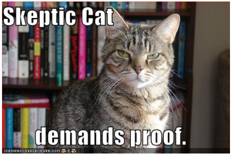

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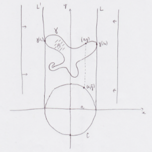

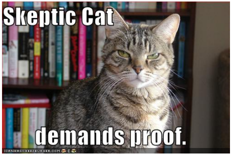

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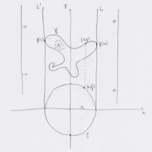

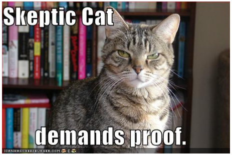

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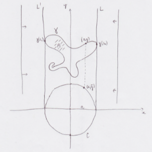
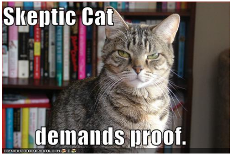

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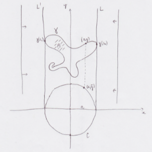

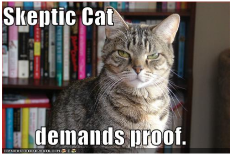

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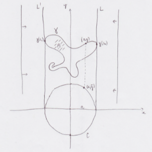
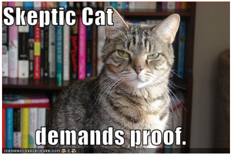

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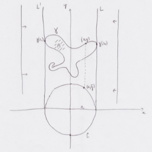
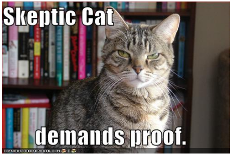
$\alpha(s) = (x(s), y(s))$ closed curve $0 \leq s \leq L$ enclosing region $A \Rightarrow L^2 \geq 4\pi A$

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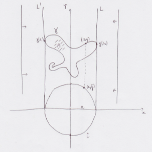
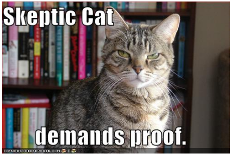

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- Gaussian isoperimetric inequality useful in Information Theory for decoding error probabilities for a Gaussian channel
- In 3-space, sphere maximizes volume while minimizing surface area—geodesic domes

Cauchy-Crofton formula

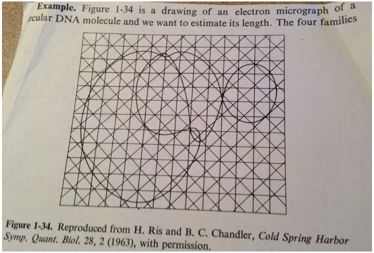
$\alpha(s)$ plane curve of length L. There are $2L$ straight lines (counted with multiplicities) which meet $\alpha(s)$

electron micrographs

Four-Vertex Theorem

$\kappa(s)$ of a simple, closed, smooth plane curve has at least four local extrema

mechanics: no polygons that can stand on only one edge (false in $\mathbb{R}^3$: Gomboc)



Example. Figure 1-34 is a drawing of an electron micrograph of a ...rcular DNA molecule and we want to estimate its length. The four families

Figure 1-34. Reproduced from H. Ris and B. C. Chandler, *Cold Spring Harbor Symp. Quant. Biol.* 28, 2 (1963), with permission.