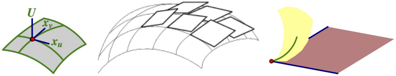
Proof of Surface Area with 1st Fundamental Form



- $\vec{x}_u \cdot \vec{x}_v = |\vec{x}_u||\vec{x}_v| \cos\theta$
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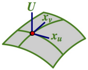

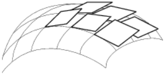
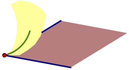

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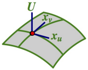

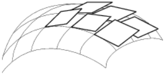
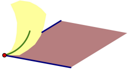

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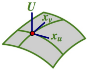

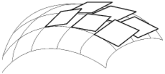
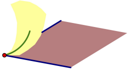

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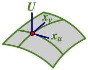

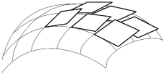
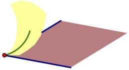

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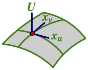

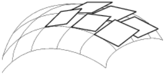
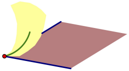

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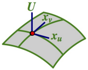

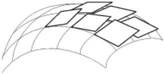
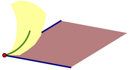

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So area $= |\vec{x}_u||\vec{x}_v| \sin\theta = \sqrt{EG - F^2}$

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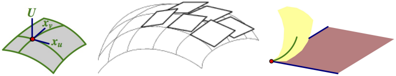

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So area $= |\vec{x}_u||\vec{x}_v| \sin\theta = \sqrt{EG-F^2}$

- Let $\Delta u, \Delta v \to 0$ and add up over the entire surface: $\int\int \sqrt{EG-F^2}dudv$

In our prior activities, for a round torus you should have calculated:

$$E = r^2 \quad F = 0 \quad \text{and } G = (R + r\cos u)^2$$

To calculate the surface area of the round donut (mmmm frosting):

a) $\int_0^{2\pi}\int_0^{2\pi} r^2(R + r\cos u)^2 \, dv \, du$

b) $\int_0^{2\pi}\int_0^{2\pi} r(R + r\cos u) \, dv \, du$

c) $4\pi^2 rR$

d) more than one holds

e) none of the above

glazed and confused

Strake Computations: I, SA, Shape Operator, II, K

$\mathbf{x}(u,v) = (v\cos(u), v\sin(u), ku)$

a) $\vec{x}_u =$

b) $\vec{x}_v =$

c) $g_{ij} =$

d) set up SA of one turn of the strake, when the inner radius is 1, the outer radius is 1.2, and the height of one turn is 10m.

e) $U =$

f) $S(\vec{x}_u) = -\nabla_{\vec{x}_u} U = -U_u =$
Then write it in the basis of $\_\vec{x}_u + \_\vec{x}_v$

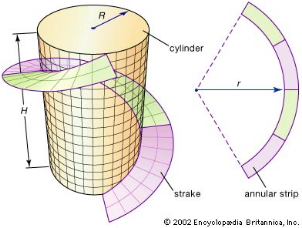
g) $S(\vec{x}_v) = -\nabla_{\vec{x}_v} U = -U_v =$
Then write it in the basis of $\_\vec{x}_u + \_\vec{x}_v$

h) $K=$

i) $l = S(\vec{x}_u)\cdot\vec{x}_u = \vec{x}_{uu}\cdot U =$

j) $m = S(\vec{x}_u)\cdot\vec{x}_v = \vec{x}_{uv}\cdot U =$

k) $n = S(\vec{x}_v)\cdot\vec{x}_v = \vec{x}_{vv}\cdot U =$



Cylinder Computations: I, SA, Shape Operator, II, K

$\mathbf{x}(\theta, z) = (r\cos\theta, r\sin\theta, z)$

a) $\vec{x}_\theta =$

b) $\vec{x}_z =$

c) $g_{ij} =$

d) set up the double integral for surface area of a cylinder with height from $z = 0$ to h

e) $U =$

f) $S(\vec{x}_\theta) = -\nabla_{\vec{x}_\theta} U = -U_\theta =$
Then write it in the basis of $\_\vec{x}_\theta + \_\vec{x}_z$

g) $S(\vec{x}_z) = -\nabla_{\vec{x}_z} U = -U_z =$
Then write it in the basis of $\_\vec{x}_\theta + \_\vec{x}_z$

h) $K=$

i) $l = S(\vec{x}_\theta) \cdot \vec{x}_\theta = \vec{x}_{\theta\theta} \cdot U =$

j) $m = S(\vec{x}_\theta) \cdot \vec{x}_z = \vec{x}_{\theta z} \cdot U =$

k) $n = S(\vec{x}_z) \cdot \vec{x}_z = \vec{x}_{zz} \cdot U =$

SA of Surface Disk on Sphere and Cone

- SA of circular disk with center at N pole of intrinsic radius r in spherical coordinates on a sphere of radius R.
  $\lim \delta \to 0 \int_{v=\delta}^{v=\frac{r}{R}} \int_{u=0}^{u=2\pi} \sqrt{EG - F^2} dudv$
- SA of circular disk of intrinsic radius r with the intrinsic center at the cone point.
  $\lim \delta \to 0 \int_{v=\delta}^{v=r} \int_{u=0}^{u=\alpha} \sqrt{EG - F^2} dudv$
- firstfundformsa.mw