In-class Timed Assessment: Surfaces

It is time for our in-class assessment, on surfaces, in order to be sure that everyone reviews some of the fundamental concepts before we move on to spacetime and relativity.

During Class

- You may make yourself some reference notes on both sides of the very small card I hand out. The mini-reference card must be handwritten. Think of the card as a way to include some important concepts, computations, or derivations that you aren't as comfortable with. You won't have room for much, so you should try to internalize as much as you can.
- You may have standalone ear plugs—no technological connections connected to the internet though.
- This assessment has an individual component as well as a component where you can work in groups.

  You work alone until I collect the individual portion and say it is "group time" and time to turn in the individual portion. Then you may continue to work alone or in groups (or a combination!). The idea is to give you opportunities to communicate course content with your peers, since this is one of ASU's main educational goals: "Successful communicators interact effectively with people of both similar and different experiences and values." The only guidelines are that each person must eventually write up and turn in their own, the only resources you are allowed to use is each other, and you should spend the time inside the classroom effectively engaging.

  If you finish the individual component early, proceed with the group component on your own until I announce group time—the idea is to have silence for a good portion of class before we switch to "group time." If you finish the entire assessment early, then you may leave early.
- Your grade will be based on the quality and depth of your responses in the timed environments. Partial credit will be given, so (if you have time) showing your reasoning or thoughts on questions you are unsure of can help your grade.

Review Suggestions

Be sure you could respond to questions on these. I want you to understand the material and I am happy to help!

Short Derivations/Proofs Be able to prove the following in the language of our class:

- That a geodesic must be a constant speed curve
- How E, F, and G and the metric equation arise from our usual definition of arc length along a curve
- $S(\vec{x}_u) \cdot \vec{x}_u = \vec{x}_{uu} \cdot U$ (i.e. how we derived l)
- That the square root of the determinant of the metric form gives the area of a flat $\vec{x}_u, \vec{x}_v$ parallelogram

These are all in our interactive videos. You should know the results of other statements too, but I won't ask you for any other complete derivations, other than the ones above.

Calculations and Interpretations By-hand computations and interpretations, like

- Finding $\vec{x}_u$ and $\vec{x}_v$ for a surface
- Using F to determine whether $\vec{x}_u$ and $\vec{x}_v$ are perpendicular ($F = 0$).
- For the plane and the cylinder (where the computations are quicker), finding the curvature vector of a curve $\vec{\kappa}_\alpha = \dfrac{T'(t)}{|\alpha'(t)|}$, a unit normal to a surface $U = \dfrac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|}$, the normal curvature $\vec{\kappa}_n = (U \cdot \vec{\kappa}_\alpha)U$ and the geodesic curvature $\vec{\kappa}_g = \vec{\kappa}_\alpha - \vec{\kappa}_n$ (what curvature is left over, if anything, for the bug to feel) for a curve.
- Sketch and label the curvature vector to a curve on a surface, normal curvature, and geodesic curvature and use that to identify if the curve is a geodesic or not
- Substituting parameters into a surface to show and identify a curve on the surface, like $x(t, t)$.
- Interpreting whether curves on a surface are geodesics via
  a) a given geodesic curvature
  b) a covering argument, if one applies
  c) geometric/physical or other arguments such as those relating to symmetry of feet
  d) geometric/physical arguments about whether the curvature vector is completely in the normal direction or not. [For example, for a circle on a surface, we know the curvature vector of any circle points to the center of the circle. Combine this with intuition about the normal to a surface to say whether the curvature vector is parallel to the normal and hence gives a geodesic (or not)]
  e) Looking at output from Maple, a visualization or numerical vectors, and interpreting it
- Finding or being presented with E, F and G and interpreting whether the Pythagorean theorem holds ($E = G = 1, F = 0$) or not
- For the plane and the cylinder (where the computations are quicker), computing I, II, K, and the shape operator
- Given K, interpreting what it tells us intuitively about the surface and being able to roughly sketch (for $K < 0, K = 0$ and $K > 0$)

Fill in the Blank/Short Answer There will be some short answer questions, such as:

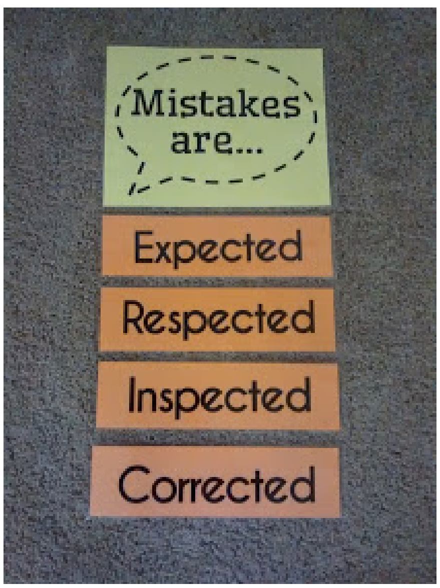
- questions similar to previous polling questions, video interactions, matching activity questions, or other activities from class where you fill in a blank instead. For instance,
  - $E =$ ______________________
  - $\vec{x}_u \cdot \vec{x}_u =$ ______________________
  - normal curvature components of a curve = _____________

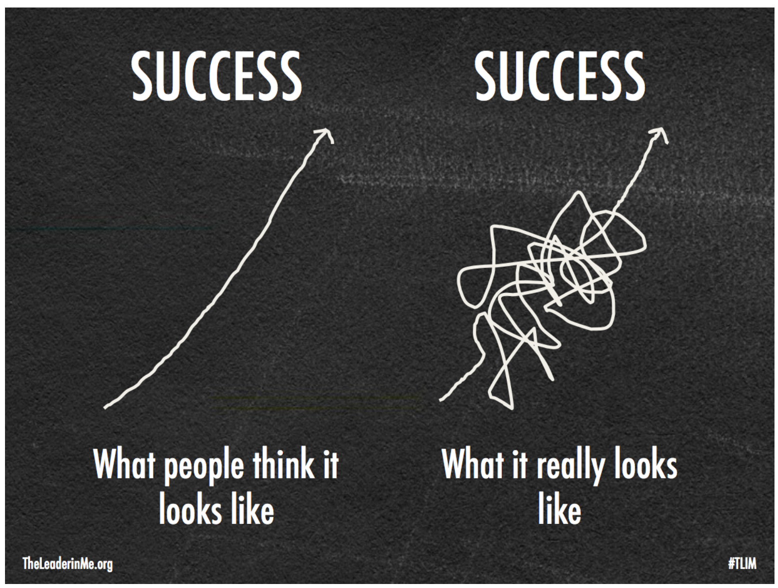
As you can see here, there is often more than one answer possible for fill in the blank questions: choose one response. Full credit responses demonstrate deep understanding of differential geometry. For instance, here you could fill in $E = \vec{x}_u \cdot \vec{x}_u$ for the first response, $\vec{x}_u \cdot \vec{x}_u = E$ for the second, and normal curvature components of a curve $= (U \cdot \vec{\kappa}_\alpha)U$ for the third, among other possible responses. On the other hand, responding with $E = E$, while a true statement, doesn't demonstrate deep understanding of differential geometry. Informal responses are fine as long as they are correct and demonstrate understanding of the material from class and homework.

- geometric properties of these 7 surfaces: cone, cylinder, hyperbolic plane, plane, sphere, strake, torus

(optional) Revision Opportunity for the Curves or Surfaces Assessment

In many real-life scenarios, such as exams in some graduate schools (e.g., in my experiences at the University of Pennsylvania), in actuarial science exams, edTPA for teacher certification, and much more, an opportunity to retake an assessment or revise part of it is allowed or even quite common. At Penn we could retake our preliminary exam after a semester had passed but the oral exam was a one-shot deal.





https://mathequalslove.blogspot.com/p/free-classroom-posters.html

https://www.leaderinme.org/blog/the-power-of-a-growth-mindset/

To encourage in-class assessments as a learning experience and to accommodate for emergencies, like missing the assessment completely, you can revise **one** of them to replace the original. There are no make-up assessments but there is flexibility in this manner.

The idea is to turn any mistakes into productive ones (or possibly even enlightening moments where you made some additional connections and hopefully have some aha moments!). Making mistakes is integral to the learning process as long as you review and understand any misconceptions, and I want to encourage and reward this. For the revision, I expect you to use online resources and get help from me. You can annotate your original graded assessment, writing your corrections on it and/or write on a separate sheet of paper. Regardless, be sure to turn back in the original too. I want you to solidify the material and I am here to help!