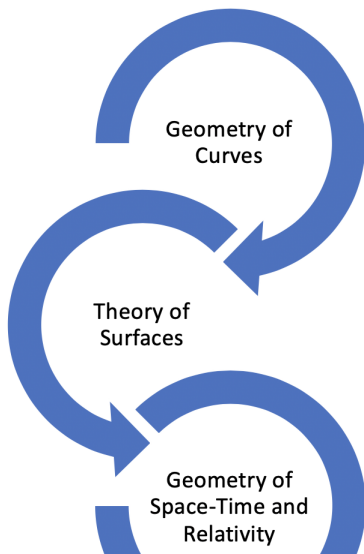


- review questions
- arc length of the tractrix $\alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$
by-hand and in Maple



1. Which of the following could represent the line between the points $(-3,2,5)$ and $(1,-2,4)$

a) $\begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix} + t \begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + t \begin{bmatrix} 4 \\ -4 \\ -1 \end{bmatrix}$

c) both of the above

d) none of the above

2. A line has

- a) velocity is $\vec{0}$
- b) acceleration is $\vec{0}$
- c) acceleration is nonzero but constant
- d) more than one of the above

3. In Euclidean space, the shortest distance path between \vec{p} and \vec{q} is the line $\vec{p} + t(\vec{q} - \vec{p})$ because:

a) No matter the geometry, we must head in the direction from \vec{p} to \vec{q} to achieve the shortest path

b) The length of the line =

$$\int_a^b \alpha'(t) \cdot \frac{\vec{q} - \vec{p}}{|\vec{q} - \vec{p}|} dt \leq \int_a^b |\alpha'(t)| \left| \frac{\vec{q} - \vec{p}}{|\vec{q} - \vec{p}|} \right| dt = \int_a^b |\alpha'(t)| dt$$

c) both of the above

d) none of the above

4. The dot product of two vectors in 3-space, $\vec{v} \cdot \vec{w}$ is

a) $|\vec{v}||\vec{w}|\cos(\theta)$ where θ is the angle between them

b) $|\vec{v}||\vec{w}|\sin(\theta)$

c) $v_1 w_1 + v_2 w_2 + v_3 w_3$, where v_i is the i th entry of \vec{v}

d) more than one of the above

5. To calculate a tangent vector and the velocity vector
- a) If the curve is parametrized by time then it is the same calculation
 - b) They are always equal
 - c) For tangent take the component derivatives with respect to the parameter in the parametrization, for velocity take the component derivatives with respect to time
 - d) more than one is true

6. Which of the following are true regarding arc length s ?

a) $s = \int |\alpha'(t)| dt$

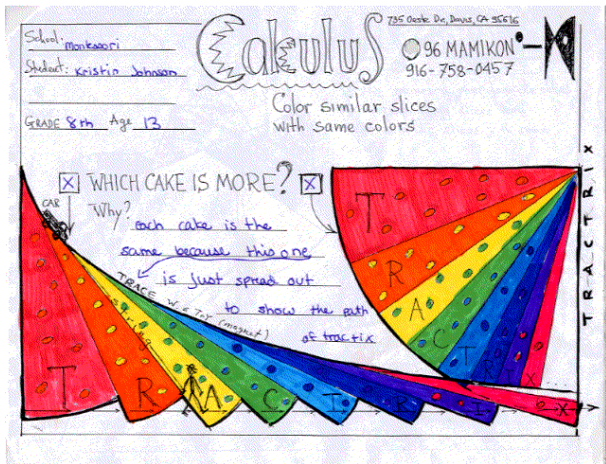
b) $s = \int \text{speed } dt$

c) a closed form solution for s may not exist, even for regular curves

d) more than one is true

1.1 & 1.2: Tractrix and Arc Length

$$\alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$$



<https://www.its.caltech.edu/~mamikon/Mont.html>

1.1 & 1.2: Tractrix and Arc Length

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} |\alpha'(\theta)| d\theta \text{ where } \alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$$



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$$\alpha'(\theta) = \left(-\sin(\theta) + \frac{\sec^2(\frac{\theta}{2})}{2 \tan(\frac{\theta}{2})}, \cos(\theta)\right)$$

1.1 & 1.2: Tractrix and Arc Length

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} |\alpha'(\theta)| d\theta \text{ where } \alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$$



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$$\begin{aligned} \alpha'(\theta) &= \left(-\sin(\theta) + \frac{\sec^2(\frac{\theta}{2})}{2 \tan(\frac{\theta}{2})}, \cos(\theta) \right) \\ &= \left(-\sin(\theta) + \frac{1}{2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2})}, \cos(\theta) \right) = \end{aligned}$$

1.1 & 1.2: Tractrix and Arc Length

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$$|\alpha'(\theta)| =$$

1.1 & 1.2: Tractrix and Arc Length

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$$|\alpha'(\theta)| = \sqrt{\left(-\sin(\theta) + \frac{1}{\sin(\theta)}\right)^2 + \cos^2(\theta)} =$$

1.1 & 1.2: Tractrix and Arc Length

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} |\alpha'(\theta)| d\theta \text{ where } \alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$$



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$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} |\alpha'(\theta)| d\theta \text{ where } \alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$$



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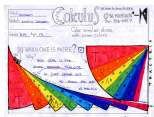
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1.1 & 1.2: Tractrix and Arc Length

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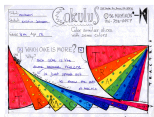


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$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} -\cot(\theta) d\theta =$$

1.1 & 1.2: Tractrix and Arc Length

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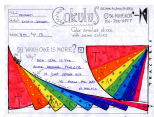


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$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} -\cot(\theta) d\theta = - \int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \frac{\cos(\theta)}{\sin(\theta)} d\theta$$

1.1 & 1.2: Tractrix and Arc Length

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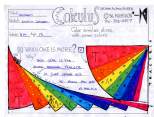
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integration by substitution $u = \sin(\theta) du = \cos(\theta) d\theta$

1.1 & 1.2: Tractrix and Arc Length

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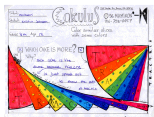
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$$= -\ln|\sin(\theta)| \Big|_{\frac{\pi}{2}}^{\frac{2\pi}{3}} = -\ln\frac{\sqrt{3}}{2} - -\ln 1$$

1.1 & 1.2: Tractrix and Arc Length

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} |\alpha'(\theta)| d\theta \text{ where } \alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$$



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$$= -\ln|\sin(\theta)| \Big|_{\frac{\pi}{2}}^{\frac{2\pi}{3}} = -\ln\frac{\sqrt{3}}{2} - (-\ln 1) = -\ln\frac{\sqrt{3}}{2} \approx .1438$$



<http://www.nerdytshirt.com/images/shirt-images/calculus-3/>

pythagorean-magnitude-t-shirt-24.jpg

