Women and Minorities in Mathematics
Incorporating Their Mathematical Achievements Into School Classrooms

African Mathematician Muhammad and His Magic Squares

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In 1792, Thomas Jefferson, who would later become the third president of the United States, said:

> Comparing them by their faculties of memory, reason, and imagination, it appears to me that in memory [the Negro] are equal to the whites; in reason much inferior, as I think one could scarcely be found capable of tracing and comprehending the investigations of Euclid; and that in imagination they are dull, tasteless, and anomalous. (Williams, 1999d)

There are many counterexamples to Jefferson's claims. The existence of mathematicians of African descent before and during the time of Thomas Jefferson, such as Benjamin Banneker and Muhammad ibn Muhammad, contradict Jefferson's assertions. In fact, Banneker, living in the U.S. at the time, responded to Jefferson's comments with a twelve-page letter (Williams, 1999b). Not only are people of African descent capable of studying and understanding known mathematics, but they have also shown imagination, creativity and mathematical ability in their investigations of original mathematics (Williams, 1999c).

There has been much written on Benjamin Banneker, including ideas for incorporating his mathematics into classrooms (e.g., Johnson, 1999; Lumpkin, 1997a; Lumpkin, 1997b; Lumpkin & Strong, 1995). Yet, not many people know about Muhammad or his mathematics and only a few people in recent times have written anything substantial about him (Gerdes, 1994; Gwarzo, 1967; Kani, 1992; Sesiano, 1994; Zaslavsky, 1999). This is unfortunate, since his mathematics can be introduced to students at many levels, ranging from elementary school children to college students in a modern algebra class. His mathematics nicely illustrates some of the connections between geometry and algebra. This article discusses Muhammad, his culture and language, his mathematics, and how to bring his research into school classrooms.

Muhammad ibn Muhammad al-Fullani al-Kishnawi

Muhammad's full name tells us his ethnic group (al-Fullani) and birthplace (al-Kishnawi). He is from the Katsina (Kishnawi) region, which is now in northern Nigeria (see Figure 1). His people, the Fullani, were mainly nomadic herders, and were one of the first African tribes to convert to Islam (Onorato, 1997). In addition to being a mathematician, Muhammad was an astronomer, astrologer, and a mystic (Gwarzo, 1967).

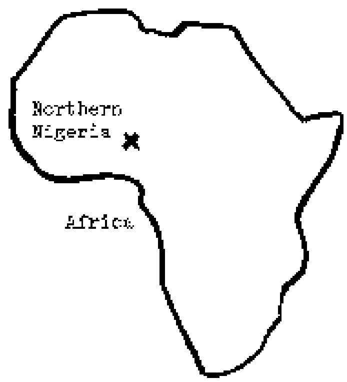

Figure 1. Sketch of Africa with an x marking Northern Nigeria.

Muhammad worked on the mathematics of magic squares. A magic square is an n x n array in which the sum of the n numbers in each row, column and diagonal add up to the same number. Magic squares have a rich history. People from many cultures and countries (e.g. Africa, China, India, Japan, France, and Germany) worked on understanding magic squares. In West Africa, where Muhammad lived until he traveled to Egypt, magic squares held spiritual importance and were found on clothing and in the designs of buildings. Today, they are still studied in relation to various fields of mathematics. Methods of constructing magic squares can make a nice classroom topic (Anderson, 2001).

Muhammad worked on odd order magic squares. The order of an n x n magic square is n, the number of rows or columns. Given n, any odd number, formulas gave the center of the n x n magic square and the sum of the entries in any row, column or diagonal (called the magic constant). Using these formulas for odd order magic squares and his ingenuity, Muhammad created methods for constructing odd order magic squares (see Activity Sheet 1) (Sesiano, 1994; Zaslavsky, 1973). A quote from Muhammad's manuscript tells us about his work ethic:

> Do not give up, for that is ignorance and not according to the rules of this art. Those who know the arts of war and killing cannot imagine the agony and pain of a practitioner of this honorable science...You cannot hope to achieve success without infinite perseverance. (Zaslavsky, 1973, p. 139)

Muhammad used East Arabic numerals (see Figure 2) and published his work in Arabic (see Figure 3).

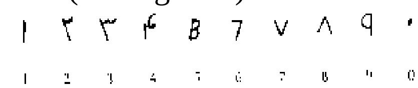

Figure 2. East Arabic numerals (Zaslavsky, 1999). Reprinted with permission of Lawrence Hill Books.

Muhammad used geometry to create new magic squares (see Activity Sheet 2). Given a 3 x 3 magic square, he constructed seven other 3 x 3 magic squares by using transformations of the square to itself. These transformations, or symmetries of the square, are also known as the dihedral group, which consists of rotations and reflections that take the square onto itself. They arise frequently in art and nature and can act to form new magic squares from old ones.

Figure 3. Muhammad's manuscript showing the eight 3 x 3 magic squares formed by rotations and reflections (Zaslavsky, 1999). Reprinted with permission of Lawrence Hill Books.

Classroom Activities

Now we offer activities designed to introduce ideas related to Muhammad's research to students. Classroom Activity Sheets 1 and 2 can be found at the end of this column and all the other activities can be found online along with teacher's notes (Greenwald, 2001). Slight modifications make these worksheets appropriate for use at a variety of grade levels.

Activity Sheet 1: Muhammad's Construction of a 3 x 3 Magic Square

This activity describes with words and diagrams the method that Muhammad developed to construct 3 x 3 magic square (adapted from Zaslavsky, 1973). In addition to this description, the student is asked to describe the construction method by creating his/her own algorithm. The algorithm can be assessed by distributing the created algorithm to another student.

Activity Sheet 2: Generating All Magic Squares of Order 3 Using Transformations

Muhammad discovered that all magic squares of order 3 could be generated by reflecting or rotating one version of the magic square. This activity leads the student through the 8 different transformations to generate all magic squares of order 3.

Properties of Magic Squares

This activity sheet has a variety of problems that address the properties of odd-order magic squares dealing with the magic constant and the middle number of the square.

Magic Square Bingo

This activity is based on a classroom activity created by students in a liberal arts mathematics class (Birch & Brewington, 2001). Hand out two copies of 3 x 3 magic square arrays to each student. Have the students cut one of the sheets into nine individual square pieces, with one number on each piece, so that the pieces will fit onto the uncut array (see Greenwald, 2001 for models to use). While demonstrating a geometric transformation, ask the class to place the pieces on top of the old magic square to indicate their new places.

Fill in the Magic Square

This activity is adapted from a worksheet created by students in a women and minorities in mathematics class (Drum & Winstead, 2001). Have students fill in the blanks in partially completed magic squares. This activity first requires students to find a completed row, column, or diagonal to add up in order to solve for the magic constant. Students must then use a combination of subtraction, guess and check, and trial and error techniques in order to fill in the blanks. Students gain an appreciation for the difficulties involved in creating magic squares, and can empathize with Muhammad's perseverance on his 11 x 11 magic square.

NCTM Standards

Investigating Muhammad's mathematics through these activities allows students to have experience with algebra by using and analyzing formulas. Algebra is also explored by having the students write algorithms. This activity gives the students a chance to communicate mathematically. Communication is one of the standards for school mathematics (National Council of Teachers of Mathematics, 2000). The mathematics of magic squares also makes a connection to geometry by using transformations that encourage the development of spatial visualization. Overall, Muhammad and his magic squares present a rich arena to explore different areas of mathematics.

References

We hope that the following activities and references are informative and enjoyable, and would be happy to mail copies of the classroom activity sheets to those without web access. Contact Dr. Greenwald at ASU Dept. of Mathematics, Boone, NC 28608 or sjg@math.appstate.edu. We welcome feedback on the use of these or similar activities in the classroom.

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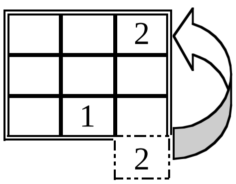
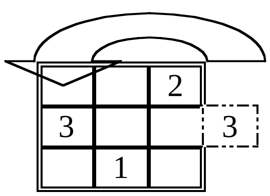
Activity Sheet 1: Muhammad's Construction of a 3 x 3 Magic Square

First Stage

You put a 1 in the middle of the lowest row. Next, look for its pair (Muhammad is referring to the cell that is diagonally down and to the right of the cell that contains 1). This should be the placement of the number 2. This particular cell, however, is not a part of the square. So put the 2 in the first position of that column.

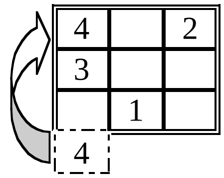


Now look for its pair and that is where the 3 should go. As in the previous case, the cell that is diagonally down and to the right of the cell with the 2 in it is no part of the square so this moves the 3 to the first position in that row.



<u>Second Stage</u>

The pair for 3 already has a number in it so you need to begin the second stage. Starting with and including the cell that holds the last number placed in the square, the 3, count three positions going down. That is the placement of the number 4. This cell, however, is not in the square. So you must move the 4 to the top position in that column.

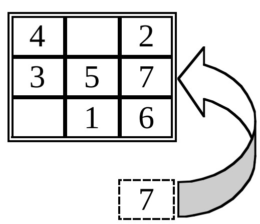


Its pair (the middle square) is where the 5 is placed. Then place a 6 as its pair.

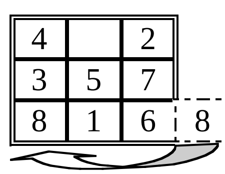
4		2
3	5	
	1	6

Third Stage

Starting with and including the cell that holds the last number placed in the magic square, the 6, count three positions going down. That is the placement of the number 7. This cell, however, is not in the square. So you must place the 7 in the second position in that column.



7 has no pair in the square. So place 8 in the first position in the lowest row.



Since 8 has no pair, put 9 in the highest position in the second column.

4	9	2
3	5	7
8	1	6
	9	

The 3 x 3 Magic Square using Muhammad's construction method is complete!

4	9	2
3	5	7
8	1	6

Muhammad's construction method for a 3 x 3 magic square can be generalized to construct a magic square of any odd order. Review the stages to construct a 3 x 3 magic square. Write the steps or an algorithm of how to construct a 3 x 3 magic square in your own words. Give your algorithm to a fellow classmate to read and see if they are able to follow your directions to create a 3 x 3 magic square.

After you have completed this algorithm write an algorithm for the construction of a 5 x 5. Exchange with a classmate to see if the steps were clear enough to follow

Activity Sheet 2: Generating All Magic Squares of Order 3 Using Geometric Transformations

Muhammad found that every possible third order magic square can be obtained by reflecting or rotating one version of the magic square. There are eight different magic squares of order 3. Problems 1-8 demonstrate the various transformations to perform.

1. The first transformation is the identity. The image of this transformation is simply the magic square itself, unchanged.

4	9	2
3	5	7
8	1	6

2. Reflect the square about the vertical line running through the middle of the square. Sketch the vertical line and reflect each number across the line. Remember that if a number lies on the line then its image is itself.

4	9	2
3	5	7
8	1	6

3. Reflect the square about the horizontal line running through the middle of the square. Sketch the horizontal line and reflect each number across the line. Remember that if a number lies on the line then its image is itself.

4	9	2
3	5	7
8	1	6

4. Reflect the square about the main diagonal that runs from top left to bottom right. Sketch the diagonal line and reflect each number across the line. Remember that if a number lies on the line then its image is itself.

4	9	2
3	5	7
8	1	6

5. Reflect the square about the main diagonal that runs from top right to bottom left. Sketch the diagonal line and reflect each number across the line. Remember that if a number lies on the line then its image is itself.

4	9	2
3	5	7
8	1	6

6. Rotate the square with the center of rotation as the center of the square and angle measure of 180^o counterclockwise. Draw the center of the square and an angle of measure 180^o.

4	9	2
3	5	7
8	1	6

7. Rotate the square with the center of rotation as the center of the square and angle measure of 90^o counterclockwise. Draw the center of the square and an angle of measure 90^o.

4	9	2
3	5	7
8	1	6

8. Rotate the square with the center of rotation as the center of the square and angle measure of 270^o counterclockwise. Draw the center of the square and an angle of measure 270^o.

4	9	2
3	5	7
8	1	6
