

Carolyn Gordon

Born: December 26,1950

Carolyn Gordon may be most commonly known for answering the question presented by Kac: Can you hear the shape of a drum? (Her answer was no.) But what else is there to Carolyn other than the math that she presented. Carolyn Gordon has an array of accomplishments from answering Kac's question to being a major active member in AMS and AWM to raising an adopted child. How did it start though?

In 1971 Carolyn graduated with a BS in mathematics from Purdue University then in 1972 she graduated from Purdue once again this time with an MS Finally, in 1979 she obtained her Ph.D. from Washington University. Throughout all of her studies she found that Riemannian Geometry was her specialty. When answering Kac's question she included Riemannian geometry into her proof.

While at Washington University Gordon met her husband David Webb (Picture to the right is Carolyn Gordon along with Husband David Webb). Together they would work along side with each other to answer Kac's question. This was no easy feat. Throughout much of the time that it took to answer Kac's question Gordon was in London doing work while David was at home working at Washington University. Large phone bills and many lonely nights put a damper on their relationship. However, Gordon was too ambitious to see things through than to let things slip away. Overtime everything worked out and their relationship grew stronger than ever. After the separation they were able to come back together to their research. During this time their house was full of nothing but little drums they had carved out to answer Kac's question.



While this was all going on Carolyn took on much more than the average human would ever try at one time. She first started her career at Israel Institute of Technology. From there she worked as an assistant Professor at Lehigh University and Washington University. Finally, she became a full professor and worked at Washington University and Dartmouth College where she presently works. While building her career in math she also spoke at many seminars and colloquia held at schools, and AWM and AMS meetings.

In the AWM she has been an active member as a panelist. The picture to the left shows the panelists of the 2001 meeting. This infers that Carolyn Gordon is a strong women's activist. Along with this too she appeals to women who want to become mathematicians. Out of the all Ph.D's awarded each year in math only 25% are awarded to women. It turns out that more then 50% of Ph.d's earned under Carolyn Gordon are given to women. This obviously shows the work that she does to improve women's chances in math.

After answering Kac's question Carolyn and David decided to start a family. They ended up adopting a child. For whatever reason they adopted it helps show the great ability of Carolyn Gordon? Her juggling of research, job, family, helping others and speaking at seminars puts her atop of the many woman and men mathematicians. The picture below shows this. She is standing with many of the most notable mathematicians of the 20th century.

Group after Noether Lecture: Top row: Sveltana Katok (Penn State), Lesley Sibner (Polytechnic Univ), Judy Roitman (Kansas), Jean Taylor (Rutgers), John Todd (Cal Tech), [???](#), Bettye Anne Case (Florida State), and Noether lecturer Krystyna Kuperberg (Auburn Univ). Bottom row: Sylvia Wiegand (Nebraska), Mary Ellen Rudin (Wisconsin), Linda Keen (CUNY), Carolyn Gordon (Dartmouth), Claire Baribaud (ETH, Zürich), Jack Quine (Florida State), Anne Leggett (Loyola Univ Chicago), Greg Kuperberg (UC-Davis)



Gordon and her Mathematical Drums

In 1911, Hermann Weyl proved that one can hear the area of a drum. Which just states that the bigger the drum, or area of the drum, the lower the tone it made. It was also later proven, by Ake Pleijel, that one could hear the length of the boundary, or the perimeter of the drum. From these results it was thought that the sound of a drum might contain enough geometric information to specify its shape uniquely. (web 1) So in 1966, the Polish-American mathematician Mark Kac asked the question, can one hear the shape of a drum? The answer is **NO**. Carolyn Gordon was a part of the team of mathematicians, that also included her husband David Webb at Washington University in St. Louis and Scott Wolpert at the University of Maryland, that came up with this answer in the spring of 1991. To understand this question and its answer, we must first know what a mathematical drum is, how we find them, and why sound-alike drums are important.

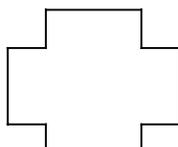
Here is an easier way to understand what Gordon was trying to accomplish. Picture in your mind

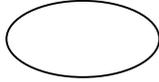
a set of drums that might be used in a band. One has a circular frame with a “skin” tightly covering the top of it. The other drum has a triangular frame with the same “skin” tightly covering the top of it. Now strike the first one with a wooden drumstick. Can you see what is happening? The skin vibrates with each strike of the stick while the circular frame stays rigid. This is also true of the triangular drum. Now say that we had a machine that can tell you the frequency of the sound that each drum vibrates at. Would you expect the frequencies to be different? I would. Two different shaped drums, two different sounds. Would you change your mind if I told you that the drums had the same area and perimeter. Probably not. But what if the machine said that the drums vibrated at the same frequency. Could you close your eyes and listen to the sounds of the drums and be able to tell which drum was making the sound, the triangular one or the circular one? No, you couldn't. This is the basis behind Gordon's work with drums. What Gordon and her team wanted to do was to see if they could find a set of drums that had the same area and perimeter but that had different shapes that sounded exactly the same. So keep these ideas of a drum in your head as we go through what is a mathematical drum.

A mathematical drum is just a shape in a plane with an interior and a boundary, such as a circle, a square, an arbitrary polygon, or just a blob surrounded by a smooth curve. (the circular and triangular drums from above) The “sounds” produced by such a drum are determined by the solutions of a partial differential equation known as the wave equation, which is used to describe any kind of wavelike phenomenon, from sound to light to water.” (web 1) (the machine that measured frequencies from above) The wave equation is $\text{Velocity} = \text{wave length} * \text{frequency}$. In essence, the motion of a vibrating membrane (that is, a drum) is governed by this equation, together with the condition that the drum will not vibrate on its boundary.

That condition is crucial. Physically, it just says that the drum is attached firmly to a frame. Mathematically, it restricts the set of solutions to the wave equation. Without some sort of boundary condition, a mathematical drum could make any sort of sound.”(web 1)

What exactly is a drum? Examples of drums are seen below.





To see how the vibrations of the drum oscillate go to <http://www.ams.org/new-in-math/cover/199706.html> (under the link that says [animation](#)).

Now that we know what a mathematical drum is, now we must ask the question, how did Gordon find her mathematical drums? Gordon used the earlier work done by Toshikazu Sunada of Nagoya University and Pierre Berard of the University of Grenoble that allowed her to find 3-dimensional drums that could then be “modded out” or smashed down into 1-dimensional drums. So after months of mathematical computation and even cutting out shapes of drums from paper, Gordon and her team came up with an example of two mathematical drums that had different shapes, which sounded alike. (the shapes in the picture below) They answered Kacs’ question with a big no.

Below is an example of how Gordon and her team accomplished this feat.



To prove whether or not you can hear the shape of a drum you must either prove that you can hear the shape of a drum for all drums or find a counter example that shows that you can’t. Knowing that it would be beyond difficult to prove that you can hear the shape of a drum Carolyn decided to show that you can not hear the shape of a drum. So she and her colleagues set out to find two different shaped drums with the same perimeter and area that would both have the same frequency. Luckily, enough some of the work had already been done. A man named Peter Buser came up with two isospectral manifolds many years ago (as seen to the above left). The only problem though was that they are bell like which make it impossible to consider them drums since drums have flat surfaces. So out of nowhere it was decided to squish

them down into flat surfaces by molding them symmetrically. The results you can see in the picture to the right. Each of the shapes that are held by Carolyn Gordon and husband/colleague David Webb has the same area and perimeter. If you were to cut one into small pieces



and place them over the picture of the other you would easily see that the cut one fits perfectly over the other.

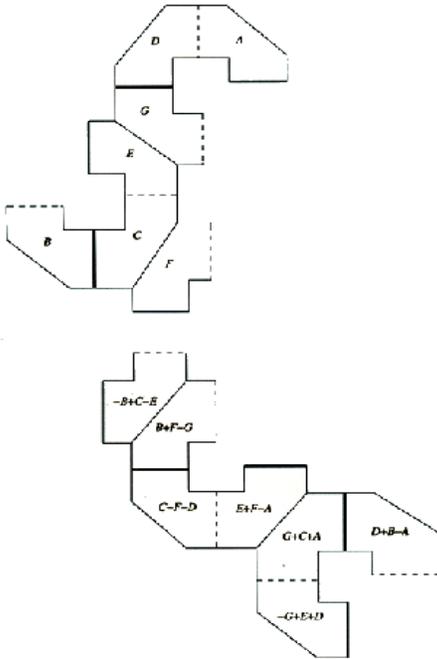
The proof that shows that the shapes have the same frequency is quite in depth. The picture to the

Figure 3. A "proof by picture" of Gordon, Webb, and Wolpert's theorem that you can't always hear the shape of a drum.

Left shows a geometric representation to the proof that proves that these shapes share the same frequencies. Below is a written description from the web page

<http://www.ams.org/new-in-math/hap-drum/hap-drum.html> of what is going on in the picture.

Proof by Picture



"Gordon, Webb, and Wolpert's first example of sound-alike drums came from a pair of curved surfaces designed by Peter Buser. Each drum consists of seven half-crosses glued together. Their streamlined proof that the drums are isospectral is based on group-theoretic principles plus Pierre Berard's "transplantation" technique, but the result is simple enough to be checked directly.

Figure 3 shows how to recombine pieces A--G of a standing wave on the seven half-crosses of the first drum into a standing wave on the second drum. For example, the combination $-B + C - E$ is formed by "flipping" pieces B and E upside down and adding them to piece C. The dark and dashed lines emphasize the required orientations. This "transplantation" is easily seen to work both ways (it can be written as an invertible 7×7 matrix). All that remains is to check that the combinations fit together smoothly and are zero on the boundary. But this can be done piece by piece. For example, $-B + C - E$ is zero on the dark boundary because $-B$ cancels C there and E was zero to begin with, while it vanishes on the dashed boundary because B is zero there and C cancels $-E$; finally, $-B + C - E$ fits smoothly with $B + F - G$ on the diagonal because $C-F$ and $E-G$ already fit smoothly together while B is zero on the

diagonal (and hence fits smoothly with its reflection).

The proof works just as well when the half-crosses are shrunk down to right isosceles triangles, and continues to work if the angles of the triangles are (simultaneously) changed. Thus one example gives rise to an entire family of sound-alike drums."

Finding the sound alike drums was important because it opened up many new mathematical areas for people to study. Finding the solution to this problem took a lot of time and effort and Gordon and her team were rewarded with the knowledge that they solved a problem which had evaded many other mathematicians for many years. This just goes to show that when you keep your nose to the grindstone, you can accomplish anything.

References

Internet references:

1. <http://www.sigmaxi.org/amsci/Articles/96articles/Gordon-Webb.html>
2. <http://www.ams.org/new-in-math/hap-drum/hap-drum.html>
3. <http://www.ams.org/ams/2001-prizes.pdf>
4. <http://www.ams.org/new-in-math/cover/199706.html>

Personal References:

1. Dr. Sarah

Thoughts on References

5. 1. <http://www.sigmaxi.org/amsci/Articles/96articles/Gordon-Webb.html>

The Internet site gives an intro to Carolyn's research with telling about Kac's question. Other than the quick intro and a few pictures that generalize what Kac's drums may look like, this Internet lacks good information about Carolyn's research.

6. <http://www.ams.org/new-in-math/hap-drum/hap-drum.html>

This site is great when looking for a little information on Carolyn's mathematical research. It gives a geometric proof which shows how the drums fit and are the same in area and perimeter. It also has great links that give pictures of the main examples they found to contradict Kac's question.

7. <http://www.ams.org/ams/2001-prizes.pdf>

This site gives a small bit of information on Carolyn's paraprofessional life. Other than that it lacks good information on her math.

8. <http://www.ams.org/new-in-math/cover/199706.html>

This site gives a good look at the drums that were created to contradict Kac's question. It also gives a link to an animated picture of the frequency and waves formed when the drums are plucked.

Dr. Sarah

Great information on Carolyn's life. Much needed info too. All other Internet sites lacked life experiences that Carolyn went through.