

Karen E. Smith

We have chosen an interesting mathematician, Karen E. Smith, to study and learn about. This will not only be interesting, but also helpful to us as future mathematics teachers. As we overview her life and the type of mathematician that she is, we will see the problems that women face in a male dominant field. Smith is known for her work in commutative algebra as well as algebraic geometry, and is at the top of her field. Trying to understand and go a little deep into her mathematics will be very beneficial. Understanding ways in which to transfer math across disciplines will be beneficial to us not only as teachers, but also as scholars in the world today.

Smith was born May 9, 1965, in Red Bank New Jersey and claims to have had the “normal” suburban childhood. She always had a love for math even as a child, however Smith was unaware that one could pursue a career in mathematics. This however changed while she was in college, where her calculus professor Charles Fefferman hinted that she look into a career in the mathematical realm. In 1987, Karen graduated from Princeton with a major in mathematics education in order to teach math to the youth of New Jersey. While teaching, Smith looked into the possibility of furthering her education and when she found out that she could receive support from the local school district while getting her masters and Ph.D. she took off for the Midwest. She attended graduate school at the University of Michigan. Under the direction of Professor Melvin Hochester, she composed her graduate thesis was on Commutative Algebra. She finished this chapter of her life in 1993.

After one year of working at Purdue University Smith moved back to Massachusetts and there became an instructor at The Massachusetts Institute of Technology or MIT. Although she liked it there, she had an urge to move back to Ann Arbor Michigan where she met her husband. She is still living there with her husband and daughter, Sanelma. She is a professor of Mathematics at the University of Michigan at Ann Arbor.

Smith is one who will not give up easily. This is obvious in her work because of she wrote a thesis in commutative algebra at the University of Michigan. Today, Smith is doing research in commutative algebra at the University of Michigan where she is also a professor. Some of her classes are Commutative Algebra, Linear Algebra, Pre-calculus, honors calculus, intersection theory, and introduction to Algebraic Geometry. Smith was not always this ambitious about school. During Smith's high school career she was not a serious student, but she did what she needed to get by. Another words, one can assume from this that she did not try her very best. However, in mathematics her views were different. She always had a love for math and was interested in learning more each day.

While continuing her mathematical career, Smith encountered many people who did not believe that she could do math. This was simply because she is a woman. Smith mentions that the sexist people were not always men. Women, as well as men, discouraged her and her work. However, the gender issues that she came upon did not hinder her, she still received a masters and a Ph.D in mathematics.

During her mathematical career she has been awarded the Ruth Lyttle Satter Prize for her work in commutative algebra. This accomplishment took place in the year 2001. This award has placed Smith as a world leader in the field of tight closure. She has also

narrowed the gap between algebraic geometry and commutative algebra through her work on tight closure.

Smith's main focus was on tight closure, the characteristic p , and determinantal variety. However, even Smith told us herself that some of these topics are more than likely too complicated for undergraduates. She did mention that we could understand characteristic p . That is what we are going to focus on in this paper. This is a small piece of what Smith actually worked on. However, we assume that she went much deeper than we could possibly dream of going.

What exactly is characteristic p ? Our first attempt of a definition of characteristic p was the definition of a characteristic ring. This defines R as a ring. If there is at least a positive integer n such that $na = 0$ for every a that is an element of R , then R is said to have characteristic n . If no such n exists, R is said to have characteristic 0 . This makes sense because zero times any number is equal to zero. In other words, this is talking about when a commutative ring with unity is said to be an integral domain if it has no zero divisors. Thus, a product is zero only when one of the factors is 0 ; that is, $a * b = 0$ only when $a = 0$ or $b = 0$. This definition, sometimes, but not always holds true for characteristic p since a characteristic p ring does not have to be an integral domain. To explain this a little deeper, a nonzero element a in a commutative ring R is called a zero-divisor if there is a nonzero element b in R such that $a * b = 0$. For example, in mod 6 , $(9 * 2) = 0$, which will be explained later on in the paper. We have just shown that neither a nor b was equal to zero, but the product was equal to zero. Therefore, a better definition of characteristic p is:

$$\text{For every } r \text{ in a ring: } r + r + \dots + r = 0.$$

In this definition, p represents the number of times you must add r in order to get back to zero. Smith refers to characteristic p as clock or modular arithmetic. This explains exactly how we knew that $(9 * 2) = 0 \text{ in mod } 6$. In whatever modular base you are in (our example is in 6) you divide the base into the product of the terms. The remainder will give you the answer of the product mod 6. Since $(9 * 2) = 18$, and 6 divides into 18 with a remainder of zero, then the answer to the problem, $(9 * 2) \text{ mod } 6$ is zero. When using clock arithmetic, you take the modular number, and that is the number of hours on the clock. In our case we have a six-hour clock. On the clock, zero equals six. The only numbers used on the clock are $\{0, 1, 2, 3, 4, 5\}$, and any other number is equal to one of those numbers. These are also called the remainders in mod 6. Another example would be to think of modulo 3. The numbers on the three-hour clock, or the remainders would be $\{0, 1, 2\}$. If one uses characteristic p definition in mod 3, then

$$0 + 0 + 0 = 0$$

$$1 + 1 + 1 = 0, \text{ and}$$

$$2 + 2 + 2 = 0.$$

Again, here you are adding them p times in order to get back to zero. This goes for all cases. If you think of modulo 4, the numbers on the four-hour clock, or the remainders would be $\{0, 1, 2, 3\}$. The idea of characteristic p is the number of times you have to add the remainders to get back to zero. For example,

$$0 + 0 + 0 + 0 = 0$$

$$1 + 1 + 1 + 1 = 0$$

$$2 + 2 + 2 + 2 = 0, \text{ and}$$

$$3 + 3 + 3 + 3 = 0.$$

It is nice that characteristic p enables us to limit the names of numbers. For instance, every number in mod 4 that is added or multiplied would equal one of those four numbers, $\{0, 1, 2, 3\}$. For instance, $(a + b) \bmod 4$ would equal the remainder of $(a + b) / 4$. An example would be $(2 + 5) \bmod 4 = 7 \bmod 4$. Therefore, $7 / 4 = 1$, with a remainder of 3. Thus, $(2 + 5) \bmod 4 = 3$.

The same is true for multiplication; $(a * b) \bmod 4$ would equal the remainder of $(a * b) / 4$. The following is an example; $(3 * 4) \bmod 4 = 12 \bmod 4$. Therefore, $12 / 4 = 3$ with a remainder of 0. Hence, $(3 * 4) \bmod 4 = 0$.

On a different note, we want to mention elliptic curves. This is not actually Smith's math, however, it is interesting to see that these curves can translate back and forth from the algebraic perspective to the geometric version. Smith did not use elliptic curves in her math, but it is a representation of how one may go from the algebraic form to the geometric form. Take a look at the curve $y^2 = x^3 - 2$. This curve has only two solutions that we know of. This equation will work only if $x = 3$ and $y = 5$, or $x = 3$ and $y = -5$. This is the algebraic form. Now, take a look at the curve, $y^2 = x^3 - 3x - 5$. One can study this for a long time and still not know if there are any integer solutions. However, if you graph this in the real and complex numbers, you will see a graph that looks like a donut. You would never think this equation would make such a neat geometrical curve.

You may be asking yourself how would you use this in real life? Well we use it every day when we tell time. Time is in mod twelve. Since we are in mod twelve, this means that twelve is equal to zero, and the number that follows will be one. So, thirteen is equal to one and fourteen is equal to two and so on. There are many other times in life

in which we use this definition. We also use this in the numbering of days in a week. We never say there are eight days in a week; we say there is a week and one day. Do you remember Fermat's Last Theorem, $x^n + y^n = z^n$. Smith did not actually work on this theorem, but it has no solutions in positive whole numbers for $n > 2$. Just think if there is an integer solution and we do everything in mod p , then we would still get integer solutions, but the numbers might be reduced. This is only a few of the real life examples for characteristic p .

There is one word to describe what type of mathematician Smith is, and that word is passionate. She does not worry about what other people say, instead she takes all of her passion and teaches others who are afraid or scared of math and turns them into people who enjoy the subject of rational and irrational numbers. However, despite her love of students she would rather be remembered for her contributions to math instead of her impact on students' lives. Smith's awards and her readiness to help others, make her a mathematician that is at the top of her field.

In conclusion, we feel that we have learned a great deal about the modular arithmetic, and challenged ourselves to push the limits of understanding. Smith is a unique mathematician in that she translates back and forth from geometry and the algebraic version. This is only something that we as future mathematicians can hope to aspire.

References:

Gallian A. Joseph. Contemporary Abstract Algebra. Lexington, Massachusetts: D.C. Heath and Company, 1994. The reference was very helpful in determining the actual definition of characteristic p . This showed the difference between a zero-divisor and an integral domain. This contradicted our first definition of char. p .

Smith, Karen. Email to the author. 6 Apr. 2001.

Smith, Karen. Email to the author. 11 Apr. 2001.

Smith, Karen. Email to the author. 16 Apr. 2001. All of these emails from Karen helped us realize what type of mathematician Karen is. We also obtained information on her life and gender issues through these emails.

Modular Arithmetic. Internet. April 23, 2001.

http://www.math.csusb.edu/faculty/susan/number_bracelets/mod_arith.html. This reference helped us to understand deeper what characteristic p was all about. The clock really helped us out with modulo.

Karen E. Smith. Internet. April 5, 2001.

<http://www.agnesscott.edu/lriddle/women/smithk.html>. This reference was an overview of Karen's life and the awards she won and type of classes she taught. This was very helpful.

Cryptography. Internet. Microsoft.com. April 23, 2001. This reference was basically just interesting. The elliptic curves lead us to relate to how Karen translated back and forth from algebra to geometry.