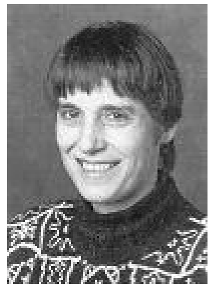
Jean Taylor

By Samantha Mathews and Stephen Howard



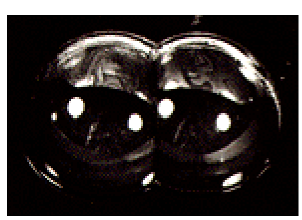
Jean Taylor was born in San Mateo, California on September 17, 1944. She later moved to Sacramento. As a child she excelled in her academics. However, her first experience with blatant sexism was in high school with her crush. He claimed it was not fair that she received higher marks than he did, for he needed better grades for his "career." This experience was only momentary though. Mrs. Taylor now claims "It inoculated me against it (sexism)." Unfortunately, she lost touch with reality and started hanging out with the bad crowd. She was known as the "ringleader" of the mischievous children.

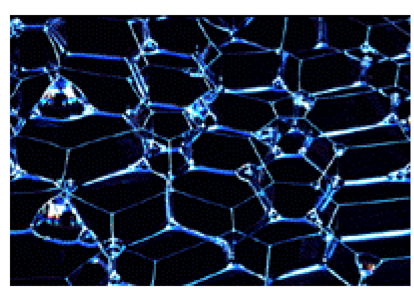
After high school she enrolled in Mount Holyoke, a women's college, in Massachusetts because she had never been east of the Rocky Mountains. She majored in chemistry and graduated Phi Beta Kappa, first in her class in 1966. Taylor, however, through her rebellious childhood, learned to question authority and was not able to do so in the chemistry laboratory at Mount Holyoke. This began to inspire her exploration into other fields of study, but she still had a love for chemistry. She later enrolled in the University of California at Berkley where she was influenced by her hiking club and her boyfriend to audit algebraic topology and differential geometry. These courses encouraged her to switch her emphases to mathematics but yet she still received her master's degree in physical chemistry in 1968. She also became very active in the protesting of the Vietnam War. Later, she moved to England shortly after her wedding of her long-time boyfriend, Frederick J. Almgren. Here, she pursued her master's degree in mathematics at the University of Warwick and graduated in 1971.

Soon after, she returned to the U.S. and attended Princeton's doctoral program in mathematics. In 1973 she received her PhD. and focused her dissertation on the topic of "Regularity of the Singular Set of Two-Dimensional Area-Minimizing Flat Chains Modulo 3 in R^3." This solved the problem on length and smoothness of soap-film triple functions curves, which had puzzled mathematicians for centuries.

The first person to work on soap films was a Belgian professor of Physics and Anatomy by the name of Joseph

Plateau. He began to study soap films in 1829, but never proved his theories. Jean Taylor's main work was that of proving Plateau's problem. Plateau's problem stated: if you start with a circle of wire that has been twisted, bent, and stretched into some new shape and dip it into soapy water and pull it out again, what kind of shape will the soap film result in? Surface tension makes the resulting soap film minimize its area while still spanning the wire frame. Taylor proved that a compound soap bubble spanning a wire frame consists of flat surfaces smoothly joined together. She also confirmed that soap bubble surfaces meet in only two ways: either exactly three surfaces meet along a smooth curve of 120 degrees or six surfaces meet at a vertex. When surfaces meet along curves or when curves and surfaces meet at points, they do so at equal angles of about 109 degrees. The pictures below illustrate the symmetry of the angles.





In nature, things tend to use the least amount of energy. For example, water does not run up hill. The least amount of energy would be to let gravity take it downhill. Nor will a ball roll up a hill on it's own. In relation to the soap bubble, a soap film wants to use the least amount of energy possible. In doing this, a soap film is created with minimal surface area.

Shape	**# of sides**	**Volume**	**Surface Area**
Tetrahedron	4	1 cubic inch	7.21 square inches
Cube	6	1 cubic inch	6 square inches
Octahedron	8	1 cubic inch	5.72 square inches
Dodecahedron	12	1 cubic inch	5.32 square inches
Icosahedron	20	1 cubic inch	5.15 square inches
Sphere	infinite	1 cubic inch	4.84 square inches

The chart above represents many possible shapes of soap bubbles, and illustrates that the minimal surface area is actually in a sphere. But why does a soap film create the thinnest layer of film? Taylor describes the nature of water and soap molecules. A non-polar layer of soap molecules that reduces surface tension covers the surface of the film. The addition of the soap to the water has important effects on the formation of the film in two ways. The surface acquires stabilizing elastic properties by stretching the layer of soap molecules. This also minimizes the thickness to basically the length of two soap molecules stacked end to end, one for each side of the film. Gravity causes the water molecules to "slide" out leaving only the soap molecules behind.

Another characteristic of soap bubbles is that there is a constant mean curvature. Mean curvature can best be explained in taking any arbitrary object and picking a point on the surface of that object. Then take a plane that runs through the point and rotates around the normal vector. (The normal vector is found by taking the second derivative at that point.) Next, take average of all the curves formed by rotating the plane. This is called the mean curvature. An analytical way to find mean curvature is best explained by the following, which is found in <u>Differential Geometry of Curves and Surfaces</u>:

The formula for finding the line of curvature is

$$K=(eg-f^2)/(Eg-F^2)=\{-\Psi'(\Psi'\varphi''-\Psi''\varphi')\}/\varphi$$

Where φ is always positive, and $\Psi'=0$ where the tangent line to the generator curve is perpendicular to the axis or rotation (normal vector) or $\varphi'\Psi''-\Psi'\varphi''=0$ where the curvature of the generator curve is zero. We can simplify this by taking a derivative and get

$$K=-\varphi''/\varphi$$

This can be used to determine surfaces of revolution of curvature. To compute principal curvatures we can make the observation that:

$$K=eg/EG, \quad H=.5(eG+gE)/(EG)$$

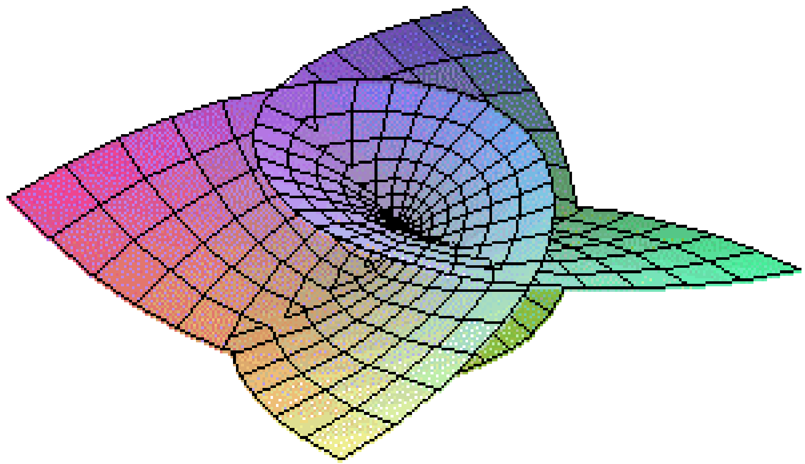
Thus the principal curvature of a surface of revolution is given by:

$$e/E=-(\Psi'\varphi)/\varphi^2=-\Psi'/\varphi$$

$$g/G=\Psi'\varphi''-\Psi''\varphi'$$

Therefore, the mean curvature is:

$$H=.5(-\Psi'+\varphi(\Psi'\varphi''-\Psi''\varphi')/\varphi)$$



The equation to the above is:

$$\mathbf{X}(u,v)=(u-(u^3/3)+uv^2,\ v-(v^3/3)+uv^2,\ u^2-v^2)$$

From <u>Differential Geometry of Curves and Surfaces</u>, we are given that the largest curvature of the above surface is

$$k_1=2/((1+u^2+v^2)^2)$$

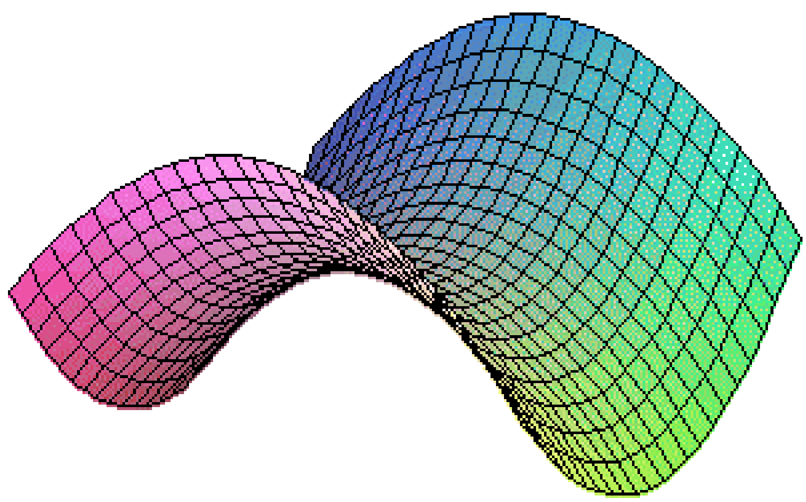
And the smallest curvature is given by the equation

$$K_2=-2/((1+u^2+v^2)^2)$$

By this we can take the average of the two and get that the mean curvature is zero. This makes sense because if you pick ant point on the surface and rotate a plane about the normal vector, you will get a positive curvature and a negative curvature. The average of the two will be zero.

Let’s look at another example that is also given in the same reference. Let’s look at the following parametric equation and its graph:

$$\mathbf{x}(u,v)=(u,v,u^2-v^2) \text{ where } u=x,\ v=y.$$



From the above we get:

$$h(u,v)=u^2-v^2$$

Now we need to take the partial derivatives of $h(u,v)$

$h_u=2u$

$h_v=-2v$

$h_{uu}=2$

$h_{vv}=-2$

$h_{uv}=h_{vu}=0$

The equation that is given for mean curvature is:

$$H=\frac{[(1+h_x^2)*h_{yy}-2h_x*h_y*h_{xy}+(1+h_y^2)h_{xx}]}{2(1+h_x^2+h_y^2)^{(3/2)}}$$

If we take the values from the example above we get:

$$H=\frac{((1+(2u)^2)(-2)+(1+(-2v)^2)(2))}{(1+(2u)^2+(2v)^2)^{(3/2)}}$$

From this we can infer that the mean curvature will be zero. This too makes sense because the figure formed by the equation is a saddle. If we pick the point at the origin we can see that in one direction the curvature is positive and in the other direction it is negative. Again the average will be zero. If $u=v$, then $H=0$.

In conclusion, Jean Taylor concentrated her studies to soap bubbles and soap films. She proved Plateau's problem and concluded that bubbles have certain properties. Some of these properties include minimal surface area and constant mean curvature.

Bibliography

http://www.scotlan.edu/Iriddle/women/jtaylor.htm

comments: this reference gave information about Taylor's life

http://www.exploratorium.edu/ronh/bubbles/bubble_meets_bubble.html

comments: this website gave excellent pictures of the angles

http://scidiv.bcc.ctc.edu/Math/Taylor.html

comments: this is another great website that gives information on her life

http://www.stanford.edu/~meadows/math/create/html

comments: this website illustrates minimal surfaces in soap film geometry

Taylor, Jean and Almgren, Frederick. *Scientific American*. "The Geometry of Soap Films and Soap Bubbles." 231, 1 pg. 82-93. July 1976.

comments: This was a fairly understandable version of Taylor's thesis and gave information on the molecular structure of the films as well and information on what exactly Taylor proved

Do Carmo, Manfredo P. *Differential Geometry of Curves and Surfaces*. Prentice Hall, Englewood Cliffs, NJ (1976). Pg 162-168.

Comments: This reference gave the equations for mean curvature as well as the examples.